Investigating Coherent Structures of Quasi Two-Dimensional Flows Subject to Electromagnetic Forces

Loutfallah M. Moubarak

Physics Department

American University of Beirut
Outline

- Introduction and Motivation

- Experimental setups, the square and cylindrical container & diagnostics: tracers, laser sheet

- Motion inside the square container: Generation of coherent vortices, secondary vortices and emergence of diagonal jets

- Motion inside the cylindrical container: stationarity and the flow dynamics

- Conclusion and Future work
2D turbulence in nuclear fusion

Fusion requirements:
- High energy due to the mass difference \( E=\Delta mc^2 \) between initial and final atoms.
- High density \( \sim 10^{20} \text{ m}^{-3} \)
- High temperature \( \sim 10 \text{ keV} \)
- Long time particles and energy confinement.

Major Fusion problem:
- High level of particles and energy losses through turbulence occurring in the perpendicular direction to the main magnetic field.
- Turbulence in magnetic fusion devices is quasi-2D because of the strong toroidal magnetic field.
- Need for a better understanding of 2D turbulence
Atmospheric and oceanic turbulence

- Large scale motion of the atmosphere is driven by pressure and temperature gradients.
- The thickness of the atmosphere or the oceans is much smaller than planets radii.
- Oceanic surface currents driven by winds.
- Frictional stress.

Atmospheric turbulence on Jupiter.
Motivation

- To reduce the loss of particles and energy by 2D turbulence in a tokamak, hence increasing its efficiency in producing energy

- To understand the atmospheric and oceanic circulation for weather predictions

- Simple experiments help to illustrate and understand large scale complex motion.
2D turbulence: Previous Experiments

Soap films

Y. Couder, 1984, Two dimensional grid turbulence in a thin liquid film, J. Physique Lett. 45, 353

Lorentz force driven experiments

J. Sommeria 1986 J. Fluid Mech. 170, 130
1- Build new experimental setups for turbulence studies and install adequate diagnostics

2- Develop numerical codes to analyze the flow: flow images, velocity field, distribution functions, statistics

3- Open up the possibility to study accurately the quasi two-dimensional properties of the flow
A new concept: Two-dimensional electromagnetic forcing to study Quasi-2D dynamics

- A transparent electrically non-conducting container is filled with conducting solution.
- Electrodes are inserted at the inner edges of the tank.
- An upward axial magnetic field created by permanent magnets or electromagnets.
The square container: From Concept to Reality

52 electrodes inserted at the 4 edges of the container, connected to a DC power supply, creating the driving electric field between consecutive electrodes separated by 2 cm.

Permanent magnets inserted from below:
Made of Neodymium.
Dimensions: 7.62 cm × 1.27 cm × 0.63 cm.
Magnetic field on top of a magnet: 250 mT.
Properties of the Magnetic field and the Electrolyte Solution

Conductive Solution: Potassium Hydroxide (KOH) dissolved in distilled water

Maximum conductivity at 27% of water mass is: $\sigma = 550 \text{ Siemens/m}$

Solution density: $\rho = 1000 \text{ Kg/m}^3$

Solution magnetic permeability: $\mu = 1.256 \times 10^{-6} \text{ H/m}$

Solution kinematic viscosity: $\nu = 10^{-6} \text{ m}^2/\text{s}$
The Cylindrical Container

- Cylinder of diameter 20 cm made of Plexiglas.
- 16 electrodes inserted at the edge => distance between two electrodes = 3 cm
- Four coils surrounding the container create the required magnetic field.

Digital Camera

Coils

*Between the upper two coils and the lower two coils, a 5 cm spacing is left for the laser sheet.*
The Magnetic field

Axial magnetic field

Bottom of the container at $z = 7$ cm

- Almost constant dependence on $r$
- Almost constant dependence on $z$

Constant magnetic field at the edge of the container as a function of the poloidal angle
Markers of the Flow:
1- Dye with uniform white light
2-Glass beads with a laser sheet

Glass beads:
• Neutrally buoyant spherical glass beads (40-60μm diameter)
• Seeding the fluid with beads and using a laser sheet follow their dynamics with time
Velocity Field Measurement (PIV) when using Glass beads + laser sheet

1. Area selection, for example 50x50 pixels
2. Filter the selected area in the two frames.
3. Select the maxima from each frame
4. Select distances that are smaller than \( D_{\text{max}} \) and greater than \( D_{\text{min}} \)
5. Get the velocity from the travelled distance

Minimum distance, \( D_{\text{min}} \), is determined from the light fluctuations without motion.

Maximum distance, \( D_{\text{max}} \), is obtained by the visual inspection of the motion.
The number of maxima in the second frame is 19. We got 9 velocity vectors for the beads of the first two frames, thus almost half of the maxima selected contribute to the velocity measurement.

After setting the values of $D_{\text{max}}$ and $D_{\text{min}}$ we measure the velocity field between consecutive frames.

Only one vector is pointing in a direction different than the others.
Validation with and without External Forcing

**Velocity field with \( I = 0 \):**
- Overlay of 30 contour plots
- Velocity vectors scattered in all directions
- Average velocity components = 0
- Noise velocity due to light fluctuations caused by dust and impurities

**Velocity field with \( I \) different than 0:**
- Overlay of 20 contour plots and the corresponding velocity field
- Almost 5% of the velocity vectors are deviated from the general trajectory of the beads
- Average velocity different from 0
Magnetohydrodynamic: Dimensionless Parameters of the Flow

**incompressible Navier-Stokes equations**

\[
\vec{v} \cdot \vec{v} = 0
\]

\[
\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho} \nabla P + \frac{\eta}{\rho} \nabla^2 \vec{v} + \frac{\sigma}{\rho} (\vec{E} + \vec{v} \times \vec{B}) \times \vec{B}
\]

**The interaction parameter** \( N \):

\[
N = \frac{\sigma (\vec{E} + \vec{v} \times \vec{B}) \times \vec{B}}{\rho \vec{v} \cdot (\vec{\nabla} \vec{v})}
\]

\[
N \sim \frac{\sigma B (\Delta V)}{\rho \nu^2} + \frac{\sigma LB^2}{\rho \nu} \sim 1650 >> 1
\]

**The Hartmann number** \( Ha \):

\[
Ha = \sqrt{\frac{\sigma (\vec{E} + \vec{v} \times \vec{B}) \times \vec{B}}{\eta \Delta \vec{v}}} \sim \sqrt{\frac{\sigma L B (\Delta V)}{\eta \nu} + \frac{\sigma B^2 L^2}{\eta}} \sim (400 - 500) >> 1
\]

**The magnetic diffusion equation:**

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma \mu} \Delta \vec{B}
\]

**The magnetic Reynolds number** \( R_m \):

\[
R_m = \sigma \mu \frac{\nabla \times (\vec{v} \times \vec{B})}{\Delta \vec{B}}
\]

\[
R_m \sim \sigma \mu L v \sim 10^{-8}
\]

**Conclusion:**

Lorenz force is the dominant force in the flow. The applied magnetic field dominates the induced magnetic field.
Square container: The Flow dynamics using Dye and a uniform white light source

- KOH Concentration: 27% of water mass
- KOH solution thickness = 1
- Driving current: 1 Ampere
- Tracer: Dye
- Camera 25 frames/second
The square container: Primary structures

When the Lorenz Force dominate the motion, the velocity is mainly caused by $\mathbf{E}\times\mathbf{B}$, hence, the two charges move in the same direction.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Primary vortices in front of the electrodes.
• (a), No current.
• (b), start-up with forces toward and away from the containers edge.
• (c) - (e), the formation of two counter-rotating vortices.
• (f) - (j), vortex deformation due to non-linear interaction with neighboring vortices.
Secondary vortices and complex motion

- Secondary vortices are formed away from the container edge.
- They result from viscous interaction of the fluid with primary vortices.
- They rotate in the counter-direction as the primary vortices.
Unexpected dynamics: The diagonal jet

A jet moves from the upper right corner to the middle of the container. The time between consecutive images is 2 sec.

- At point 2, the outward force generates the jet (indicated by the black arrow).

- The jet interacts with large vortices.

- The jet is driven to the middle of the container by the large vortices.

\[ \vec{j} \times \vec{B} \]
The emergence of vertical jets

Observations:
- Secondary vortices
- Two jets heading from top to bottom in the same direction.

Interpretation:
The magnets position below the container and the magnetic field distribution reflecting a strong magnetic field gradients between adjacent drawers could be the reason behind the observation of the vertical jets.
Jet formation as a function of time

- **Secondary vortices**
  - t = 2-8sec

- **Primary vortices**
  - t = 30-40sec

- **Secondary vortices merging, Large vortex**
  - t = 10-20sec

- **Diagonal jet**
  - t = 116-122sec

- **Jets**
  - t = 30-40sec
The cylindrical container: Stationarity

We calculate the average kinetic energy per frame:

\[
\overline{E}_{frame} = \frac{1}{2N} \rho \Sigma \left( v_x^2 + v_y^2 \right)
\]

\(N\) is the total number of maxima in the area chosen per frame.

- Tracer: glass beads; Lighting source: Laser sheet.
- Start recording for almost 9-10 seconds, then turn on the current, FPS = 4.
Evolution of the flow near the edge

- The energy increases in the \( x \) direction after turning the current on.
- The same time dependence in \( E_x \) and \( E_y \) reflects a diagonal path of the beads, caused by a vortex formation.
- The stationary state is reached after \( \sim 10 \) sec.

(a), \( t = 13-21 \) sec (b), \( t = 40-48 \) sec.
(c), \( t = 53-64 \) sec (d), \( t = 69-80 \) sec.
(e), \( t = 83-91 \) sec (f), \( t = 101-107 \) sec.
Evolution of the flow around the middle

- The energy in the $x$ and $y$ directions are of equal magnitudes. This is an indicator for the diagonal trajectories followed by the beads.
- The persistence of the diagonal trajectories indicates that a large vortex and the area chosen covers a small part of this vortex.
- Stationary state reached after ~65 seconds.

(a), $t = 93-107\text{sec}$
(b), $t = 107-120\text{sec}$
(c), $t = 120-133\text{sec}$
• The coherent structures are distributed around the edge of the container.

• A large scale vortex is observed in the middle.

• Kurtosis = 8.5, 8

• Skewness = 0.5, -0.4

The velocity distributions are non-Gaussian.
The Probability Distribution Function of the velocity at the edge

- Kurtosis = 4.4, 5.2
- Skewness = -0.02, 0.3

- The projection of the velocity vector in the $x$ direction gives positive and negative values.
- The projection in the $y$ direction gives only negative values.
Conclusion

- We constructed several experiments to understand the properties of quasi-2D coherent structures.
- We also developed the adequate diagnostics to evaluate both the quantitative and qualitative nature of the flow’s motion.
- We developed a numerical code to determine the two components of the velocity and initiated the determination of the statistical properties.
- We verified the onset of primary vortices and described their main properties.
- The existence of secondary vortices is put forward making the flow’s dynamics more complex far from the edge.
- The onset of a diagonal jet was studied in detail, a phenomena linked to the geometry of the square container and the magnetic field.
Future work

- Build a cylindrical container with electrodes separation equal to 1cm. The size of the edge coherent vortices becomes smaller.
- Build a square container of smaller size to be used with electromagnets generating homogeneous axial magnetic field in the flow.
- Increase the intensity of the magnetic field and reduce that of the electric current.
- Perform experiments with different heights to investigate the effect of the depth of the KOH solution layer on the dynamics.
- Perform parametric dependence on the current and/or the magnetic field strength.
Square container drawback

1. Problems:
   - Existence of magnetic field inhomogeneity in the container specifically at the edges.
   - Existence of corners in the container with probably different dynamics (jet)

2. Solutions:
   - Replacing the permanent magnets below the container with coils (electromagnets) surrounding the container (Constant magnetic field)
   - Changing the shape of the container from square to circular. (No corners, no jets)
The vertical velocity profile, Hartmann layer

Goal: Find the variation of the horizontal velocity with height, in order to deduce the thickness of the boundary layer in the presence of a magnetic field.

Assumptions: No-slip condition, \( v(z=0) = 0 \), and \( v(z=w) = v \) (stream velocity); \( w \) is the thickness of the solution. Consider a steady state flow.

\[
\vec{v} = v(z)\hat{i}; \quad \vec{E} = E\hat{j}; \quad \vec{B} = B\hat{k}
\]

The \( x \) component of the momentum balance equation becomes:

\[
\rho \frac{\partial v(z)}{\partial t} = -\frac{\partial p}{\partial x} + \eta \frac{\partial^2 v(z)}{\partial z^2} + \sigma EB - \sigma v(z)B^2
\]

which, for a steady state, becomes

\[
\frac{\partial^2 v(z)}{\partial z^2} - \frac{\sigma B^2}{\eta} v(z) = \frac{1}{\eta} \frac{\partial p}{\partial x} - \frac{\sigma EB}{\eta}
\]

This is a second order inhomogeneous differential equation. The homogeneous equation is:

\[
\frac{\partial^2 v(z)}{\partial z^2} - \omega^2 v = 0
\]

\[
\omega^2 = \frac{\sigma B^2}{\eta}
\]
Velocity Field Measurement (PIV)

Experiment with glass beads $\rightarrow$ velocity measurement $\rightarrow$ numerical code.

Steps Followed:

I-Removing noisy spikes and filtering: (Consider one frame)

1. Area selection, 100*100 pixels (a).

   Presence of dust and impurities

2. Frames cleaning: Frames subtraction, light intensity reduction. (b) Filtering, 2D filter

3. Frame Interpolation: 2D cubic interpolation (c).

4. Threshold: First maxima selection. Threshold $\approx$ std(light intensity). The maxima below the threshold are removed (non negatives). (d)

In (d), after the threshold application, several maxima are gathered next to each others $\rightarrow$ Further cleaning of the frame
The remaining corners

The opposite corners displayed similar dynamics. At two opposite corners, two jets are observed, at the remaining two corners, the jets are absent.
Square container, jets collision, large scale vortices

1- Black arrows indicate the jets direction before collision and red arrows indicate jets direction after collision

2- Large scale vortices are observed around the center of the container. They result from the interaction between the jets the primary and the secondary vortices

3- Second row of vortices (delimited with black dashed rectangles)
Toward a better understanding of the diagonal jet

Motivation: The dynamics of the flow contribute to the formation of the jet. A possible merging of certain secondary vortices leads to large vortices that interact with the jet.

Goal: Record the flow from the moment the current is turned on, in order to follow the dynamics from rest *(in contrast with the previous experiment)*.

- If there is a jet, it will be tracked from the moment it starts.
- The tracer used is the glass beads. The laser sheet illuminates the flow.
- Start the recording with I = 0 A, then after 10 seconds turn on the current while the recording continues.
- Perform contour plots for a large area near one corner at different time intervals.
Interpretation of the vertical jets

Possible interpretations:

1. *The gradB velocity* \( \approx \frac{\vec{B} \times \nabla \vec{B}}{B^3} \)
   - Consider the space between the first two drawers. The gradB velocity should be pointing in the \( y \) direction.
   - The presence of positive and negative gradients lead us to think that two adjacent jets should be moving in opposite directions. However, we only have 1 jet.

2. *Maxwell’s equation*:
   \[
   \nabla \times \vec{B} = \mu_0 \vec{j}
   \]
   \[
   \vec{j} = \sigma (\vec{v} \times \vec{B})
   \]
   \[
   v_x = \frac{1}{\mu_0 B} \frac{\partial B}{\partial x}
   \]

   The jets are moving perpendicular to the \( x \) direction, in contrast with the experimental observation.
On average, 0.5% of the beads get in and out of the laser sheet for 50 frames.