

Seminar by Dr. Vladimir Lotoreichik (Nuclear Physics Institute, Czech Academy of Sciences)

Title: Optimization of low Robin eigenvalues on 2-manifolds, exterior domains, and unbounded cones

Date and Time: February 4, 2021 at 3:00pm

Registration Link:

<https://aub.webex.com/aub/onstage/g.php?MTID=e37b8ccde6e8447369bab7caac3a0ef37>

Abstract:

In this talk we will overview several recent results on the shape optimization for the low eigenvalues of the Robin Laplacian in different geometric settings with a negative boundary parameter.

First, we will focus on optimization for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on a compact, smooth, simply-connected, two-dimensional manifold with C^2 -boundary of a fixed length [2]. The eigenvalue is optimized in the sub-class of manifolds, for which the Gauss curvature satisfies the pointwise inequality $K \leq K_0$ for a fixed constant $K_0 \geq 0$. This constraint on the curvature naturally enters into the problem. Our main result in this setting can be concisely formulated as follows: *the geodesic disk on the manifold of the constant Gauss curvature K_0 is a maximizer*.

Second, we will discuss the results on optimization of the low eigenvalues of the Robin Laplacian with a negative boundary parameter in the exterior of a bounded open set with C^2 -boundary. Namely, we will optimize the lowest eigenvalue in the exterior of a generic bounded open set in two dimensions under a constraint on the perimeter and in the exterior of a bounded convex open set in higher dimensions under a constraint on the mean curvature of the boundary related to the Willmore energy [3]. Moreover, in the exterior of a bounded planar convex set we are able to optimize the second eigenvalue under a suitable constraint on the curvature [1]. In all the settings the maximizer is given by the complement of a disk or a ball depending on the dimension.

Finally, we will discuss a result [2] on the optimization of the lowest Robin eigenvalue on an unbounded three-dimensional Euclidean cone Λ with a C^2 -smooth, simply-connected cross-section $\Lambda \cap \mathbb{S}^2$ of a fixed perimeter. We prove that the cone with a circular cross-section is an optimizer.

REFERENCES

- [1] P. Exner and V. Lotoreichik, Spectral optimization for Robin Laplacian on domains admitting parallel coordinates, *to appear in Math. Nachr.* [arXiv:2001.02719](https://arxiv.org/abs/2001.02719).
- [2] M. Khalile and V. Lotoreichik, Spectral isoperimetric inequalities for Robin Laplacians on 2-manifolds and unbounded cones, [arXiv:1909.10842](https://arxiv.org/abs/1909.10842).
- [3] D. Kréjčířik and V. Lotoreichik, Optimisation of the lowest Robin eigenvalue in the exterior of a compact set, II: non-convex domains and higher dimensions, *Potential Anal.* 52 (2020), 601–614.

