AUB–CIMPA 2023–Complex Analysis and Geometry
June 5 till 16, 2023

1 Abstracts of the Lectures

1.1 Approximation Theory in Complex Analysis

Purvi Gupta, Indian Institute of Science, Bangalore, India
Abstract: Being able to approximate functions from a given class by 'nice' functions such as polynomials or rational functions is a powerful and time-tested tool in analysis. Along with the pioneering works of Weierstrass and Runge, approximation theory in (one variable) complex analysis is driven by well-known results due to Mergelyan, Vitushkin, Arakelian, and others. Not only are these results extremely useful in themselves, attempts to generalize them to broader settings have led to the development of new techniques and notions in mathematics. On the one hand, approximation theory in several complex variables borrows techniques from a wide range of topics such as functional analysis, potential theory, singular integral representation theory, and PDEs (specifically, the so-called dbar method). On the other hand, this theory has many applications, particularly to embedding problems, giving this subject a geometric flavour.
In this course, we will first review the proofs of a selection of one-variable results, focusing on abstract (rather than constructive) techniques since these are more adaptable to higher dimensions. We will then discuss the obstructions in generalizing these results to several complex variables, as a consequence of which we will encounter different notions of convexity. At this stage, we will emphasize certain topological aspects of this theory. Towards the end of the course, we will mention some recent developments in this subject.

1.2 Introduction to Holomorphic Dynamics in Several Complex Variables

Jasmin Raissy, Université de Bordeaux, France
Abstract: In this course we will study the dynamics of a holomorphic endomorphism of a complex manifold of dimension at least 2 both from a global and local point of view. From the global point of view, we shall focus on the Fatou set, that is the place over which the dynamics is stable, and discuss some of its geometric properties. From the local point of view will will focus on the local dynamics near a fixed point, in particular when the differential of the map at the fixed point is the identity. We shall then study how to use information on the local dynamics in the study and classification of the Fatou set.

1.3 Nonlinear Fourier Transforms and Applications to Complex Analysis

Shiferaw Berhanu, Temple University, Philadelphia PA, USA

Abstract: This mini-course introduces a class of nonlinear Fourier transforms known as FBI (for Fourier, Bros, and Iagolnitzer) transforms that characterize local and microlocal smoothness, analyticity, and Gevrey regularity of functions. Applications of these transforms to the study of the regularity of solutions of systems of first order linear and nonlinear partial differential equations will be presented. In particular, applications to the holomorphic extendability of CR functions will be discussed.

1.4 Vanishing Cycles in Holomorphic Foliations by Curves and Foliated Shells

Sergey Ivashkovich, Université de Lille, France

Abstract: We shall give in our lectures an introduction to the theory of holomorphic foliations. Any preliminary knowledge of foliation theory is not required but some knowledge of complex analysis of several complex variables is assumed. The supposed length of the course is 6 lectures. The material will be organized in the following three chapters:

Chapter 1. Foliations on Real Manifolds.
1.1. Poincaré–Bendixson Theorem. 1.2. Real analytic foliations and Haefliger theorem. 1.3. Vanishing cycles and foliated currents. 1.4. Compact leaves and Novikov's theorem.

Chapter 2. Singular Holomorphic Foliations.
2.1. Smooth and singular holomorphic foliations. 2.2. Reduction of singularities. 2.3. Baum–Bott index and formula. 2.4. Non-existence of exceptional minimal sets in codimension one holomorphic foliations on $\mathbb{P}^n$ for $n \geq 3$. 2.5. Relation to Levi flat hypersurfaces.

Chapter 3. Vanishing Cycles and Foliated Shells.
3.1. Simultaneous uniformization and Poincaré domains. 3.2. Vanishing ends and completed leaves. 3.3. Extension of meromorphic mappings after a reparametrization and foliated shells. 3.4. Imbedded cycles and shells. 3.5 Examples.

1.5 On Nondegeneracy Conditions for the Levi Map in Higher Codimension

Francine Meylan, University of Fribourg, Switzerland

Abstract: Let M be a real submanifold of $\mathbb{C}^N$, $p \in M$ and $\mathrm{Aut}(M, p)$ be the stability group of M at point p that is the set of (germs of) biholomorphisms F fixing p and such that $F(M) \subseteq M$. For a real hypersurface in complex dimension 2, H. Poincaré initiated the study of the stability group by looking at Taylor series expansion: the condition $F(M) \subseteq M$

means that $\rho(F(z,w))|M = 0$ where ρ is a defining function of M and this equation gives some constraints on the Taylor series coefficients of F. The process was carried out much later in a significant manner by Chern and Moser (1974). They proved that if M is a real-analytic hypersurface through a point $p \in \mathbb{C}^N$ with non-degenerate Levi form at p and if F and G are two germs of biholomorphic maps preserving M with the same 2-jets at p then they coincide. After a historical introduction including Cartan's Uniqueness Theorem (1931) and its consequences (the Riemann mapping Theorem fails in higher dimension!) we will present various definitions of nondegeneracy of the Levi map for real submanifolds of higher codimension in CN compare them by giving many examples and discuss the generalization to higher codimension of the above 2-jet determination theorem established by Chern and Moser.

1.6 Complexes of Differential Operators

Michael Eastwood, University of Adelaide, Australia
Abstract: The classical differential operators of grad, curl, and div in Euclidean 3-space can be substantially generalised to obtain the de Rham complex on an arbitrary smooth manifold. In 1864 Saint-Venant, motivated by continuum mechanics, introduced another complex of differential operators in Euclidean 3-space including a second order operator, often called the double-curl. This complex also admits a substantial generalisation in the realm of projective differential geometry. In 1990 Rumin introduced a complex of differential operators on a contact manifold, effectively replacing the de Rham complex but including a second order operator. There are many such variations and this course will describe several of them and how they are linked. We shall start from scratch. The only real ingredient is the equality of mixed partial derivatives.

2 Abstracts of Talks

2.1 Overview Several Complex Variables

Rafael B. Andrist, American University of Beirut, Lebanon, and University of Ljubljana, Slovenia
Giuseppe Della Sala, American University of Beirut, Lebanon
Abstract: We give an overview of the research in Several Complex Variables, with a focus on the historical development in the 19th and 20th century.

2.2 Squeezing functions and type of boundary points

Houcine Guermazi, Université de Monastir, Tunisia
Abstract: I want to talk about the link between the D'Angelo type of a boundary point of a bounded domain in $\mathbb{C}^d$ and the asymptotic behavior of the squeezing function:

- link between the asymptotic behavior of the squeezing functions corresponding to ball and strictly pseudoconvex boundary points.
- link between the asymptotic behavior of the squeezing functions corresponding to polydisk and infinite D'Angelo type boundary points.

2.3 Degenerate complex Monge–Ampère equations

Mohammed Salouf, Chouaib Doukkali University, El Jadida, Morocco
Abstract: Let (X, ω) be a compact Kähler manifold. A Kähler–Einstein metric on X is a Kähler metric ω that satisfies $\mathbf{Ric}(\omega) = \lambda\omega$. Here, $\mathbf{Ric}(\omega)$ denotes the Ricci curvature of ω. When $\lambda = 0$, it was shown by E. Calabi in 1954 that constructing Kähler–Einstein manifolds entails solving a PDE of Monge–Ampère type. In my talk, I will focus on Monge–Ampère equations of the form

$$(\omega + dd^c u)^n = \mu$$

where μ is a probability measure vanishing on pluripolar sets.

2.4 Squeezing function

Abdelwahed Chrih, Université de Monastir, Tunisia
Abstract: We define a squeezing function on bounded domains of $\mathbb{C}^n$ and we describe its properties such as the existence of an extremal map, continuity, stability.