Supplementary Material for 'Modeling of Economic and Financial Conditions for Nowcasting and Forecasting Recessions: A Unified Approach'

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Appendix A The data set

Table A.1: Set of Economic Variables: Series labels and their descriptions

Series Label	Description
ip	Industrial production index
import	Import quantity index
export	Export quantity index
retails	Retail sales volume index
pmi	Purchasing manager index
empna	Total employment less agricultural employment
traserv^q	Trade and services turnover index - quarterly
traserv^m	Trade and services turnover index - monthly

Table A.2: Set of Economic Variables: The transformations, adjustments, periods, frequencies and sources of coincident series

Series Label	T	Start	End	SA&NSA	Freqency	Source
ip	3	1986:7	2018:9	SA	M	TURKSTAT[1]
import	3	1997:1	2018:9	SA	M	TURKSTAT
export	3	1997:1	2018:9	SA	M	TURKSTAT
retails	3	2010:1	2018:9	SA	M	TURKSTAT
pmi	3	2011:1	2018:10	SA	M	ICI[2]
empna	3	2005:1	2018:8	SA	M	TURKSTAT
traserv^q	3	2005:I	2018:II	SA	Q	TURKSTAT
traserv^m	3	2009:1	2018:9	SA	M	TURKSTAT

Note: T indicates the transformation of variable to ensure stationarity (1=level, 2=first difference, 3=first difference of logarithm). SA and NSA denote the adjustment to remove potential seasonality from series, where SA stands for Seasonally Adjusted or NSA for Not Seasonally Adjusted. M and Q denote frequency of the series, where M stands for Monthly and Q for Quarterly.
[1] TURKSTAT : Turkish Statistical Institute
[2] ICI : Istanbul Chamber of Industry

Table A.3: Set of financial variables: Series labels and their descriptions

Series Label	Description
FXRes	Real Central Bank's Gross Foreign Exchange Reserves
goldres	Central Bank's Gross Gold Reserves
m1	Money Stock : M1
m2	Money Stock : M2
m3	Money Stock : M3
rm1	Real Money Stock : M1
rm2	Real Money Stock : M2
rm3	Real Money Stock : M3
bist100tra	Stock Exchange Trading Volume on the Istanbul Stock Exchange
rbist	Real Stock Price Index on the Istanbul Stock Exchange
VOL	Volatility on the Istanbul Stock Exchange 100
P/E	Price-Earning Ratio on the Istanbul Stock Exchange 100
liv	Cost of Living Index for Wage Earners
ppi	Producer Price Index
Conf	Real Confidence Index
embi	JP Morgan Emerging Markets Bond Index-Turkey
EMBI-Tr	Spread between JP Morgan Emerging Markets Bond Index-Turkey and 1-month Interest Rate on deposits
MSCIem	MSCI-Emerging Market Index
TETS	Spread between the 3-month Interest Rate on deposits and 3-month London Interbank Offered Rate
TermS	Spread between the 1-year and 1-month Interest Rate on Deposits
intbnk	Interbank Overnight Interest Rate
int1m	Interest Rate on Deposits - up to 1 month
int3m	Interest Rate on Deposits - up to 3 months
int6m	Interest Rate on Deposits - up to 6 months
int1y	Interest Rate on Deposits - up to 1 year
int1y_m	Interest Rate on Deposits - up to 1 year and more
discount	Discount Rate
TAuc	Treasury Auction Rate
cds	Credit Default Swap for Turkey 5-year Bond
dbeta	Downside Beta-Bist100 and MSCI World Index
exrate	Average USD-TRY Nominal Exchange Rate
exratecpi	CPI-based Effective Real Exchange Rate (base year=2003)
curac	Current Account Balance/ Nominal GDP (in $)
finac	Balance Of Payments-Financial Account/Nominal GDP (in $)
intdebt	Real Internal Debt Stock
Cred	Banking Sector Credit Loans
bnksec	Banking Sector-Securities at fair value through profit/loss, Securities available for sale, and securities to be held till maturity-real value
elpro	Gross Electricity Production
bullp	Gold Price Growth Rate (in $)
euribor3m	Euro Interbank Offered Rate-3 month
libor3m	London Interbank Offered Rate-3 month
efunr	Effective Federal Funds Rate
tedsprd	TED Spread: Spread between 3-month US Treasury bill and 3-month LIBOR
vix	CBOE Volatility Index: VIX growth rate

Table A.4: Set of financial variables: The transformations, adjustments, periods, frequencies and sources of coincident series

Series Label	T	Start	End	SA&NSA	Source
FXRes	3	1990:2	2018: 9	NSA	CBRT[3]
goldres	3	1990:2	2018:10	NSA	CBRT
m1	3	1990:1	2018:10	SA	CBRT
m2	3	1986:2	2018:10	SA	CBRT
m3	3	1986:2	2018:10	SA	CBRT
rm1	3	1990:1	2018:10	SA	CBRT
rm2	3	1986:2	2018:10	SA	CBRT
rm3	3	1986:2	2018:10	SA	CBRT
bist100tra	3	1998:2	2018:10	NSA	Bloomberg
rbist	3	1986:3	2018:10	NSA	Bloomberg
VOL	3	1988:2	2018:10	NSA	ISE[4]
P-E	2	1988:2	2018: 9	NSA	ISE
liv	3	1996:2	2018:10	SA	CBRT
ppi	3	1994:2	2018: 9	SA	CBRT
Conf	3	1988:1	2018:10	NSA	CBRT
embi	3	1999:8	2018:10	NSA	World Bank
EMBI-Tr	2	1996:6	2018: 9	NSA	World Bank
MSCIem	2	1996:6	2018: 9	NSA	World Bank
TETS	2	1996:6	2018: 9	NSA	World Bank
TermS	2	1996:6	2018: 9	NSA	World Bank
intbnk	2	1990:1	2018:10	NSA	OECD Statistics
int1m	2	2002:8	2018: 9	NSA	TDM[5]
int3m	2	2002:8	2018: 9	NSA	TDM
int6m	2	2002:8	2018: 9	NSA	TDM
int1y	2	2002:8	2018: 9	NSA	TDM
int1y_m	2	2002:8	2018: 9	NSA	TDM
discount	2	1964:1	2018: 9	NSA	IFS[6]
TAuc	3	1994:6	2018:10	NSA	TREASURY
cds	3	2000:11	2018:10	NSA	Bloomberg
dbeta	2	1987:1	2018:10	NSA	Thomson One
exrate	3	1990:1	2018:10	NSA	CBRT
exratecpi	3	1994:1	2018: 9	NSA	BIS[7]
curac	1	1992:1	2018: 9	SA	CBRT
finac	1	1992:1	2018: 9	SA	TREASURY
intdebt	3	1998:1	2018: 9	NSA	TREASURY
Cred	3	1998:1	2018:10	NSA	CBRT
bnksec	3	1986:1	2018: 9	NSA	CBRT
elpro	3	1999:1	2018:10	SA	TETC[8]
bullp	3	1998:1	2018:10	NSA	CBRT
euribor3m	2	1999:1	2018:10	NSA	FRED[9]
libor3m	2	1986:2	2018:10	NSA	FRED
efunr	2	1954:8	2018:10	NSA	FRED
tedsprd	3	1986:1	2018:10	NSA	FRED
vix	3	2004:2	2018:10	NSA	FRED

Note: T indicates the type of transformation of variables to ensure stationarity (1=level, 2=first difference, 3=first difference of logarithm). SA stands for Seasonally Adjusted or NSA for Not Seasonally Adjusted. All series are at monthly frequency. Series at higher frequencies are converted to monthly frequency by using daily averages. The volatility of the market index BIST100, *bistvol*, is the realized volatility computed using the daily returns of the index for in the corresponding month. The downside beta for Turkey, *dbeta*, is computed using the market index BIST100 and MSCI World Index. For further details, see Bawa and Lindenberg (1977).

[3] Central Bank of Republic of Turkey
[4] Istanbul Stock Exchange (Borsa Istanbul)
[5] Turkey Data Monitor
[6] International Financial Statistics
[7] Bank for International Settlements
[8] Turkish Electricity Transmission Company
[9] Federal Reserve Bank of St. Louis Economic Database

Appendix B Econometric Model

In this section we provide details about the econometric model. In the next section we discuss Bayesian inference of the model parameters in detail. The econometric model is as follows

$$\begin{aligned}
y_{i,t} &= \lambda_i f_t + \varepsilon_{i,t} \\
\psi(L)\varepsilon_{i,t} &= \epsilon_{i,t} \quad \epsilon_{it} \sim t(0, \nu, \sigma^2_{i,t}) \\
\sigma^2_{i,t} &= \sigma^2_{i,1}\,\mathbb{I}\,[t \leq \tau] + \sigma^2_{i,2}\,\mathbb{I}\,[t > \tau] \;\text{ for } i = 1, \ldots, N \\
f_t &= \alpha_{\mathbb{S}_t} + \Phi f_{t-1} + \eta_t \quad \eta_t \sim N(0, \Sigma) \\
S_{2,t-\kappa_{S_{1,t}}} &= S_{1,t}.
\end{aligned} \tag{B.1}$$

For the autoregressive dynamics of the idiosyncratic factors, we use an AR(3) specificication for the coincident variables. For the financial variables, we assume that the idiosyncratic factors are temporally independent. The resulting model can be cast into a state-space form as

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{H}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t & \boldsymbol{\varepsilon}_t|\xi_t \sim N(\mathbf{0}, \mathbf{R}_t) \\
\boldsymbol{\beta}_t &= \boldsymbol{\alpha}_{\mathbb{S}_t} + \mathbf{F}\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t & \boldsymbol{\eta}_t|\xi_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_\mathbf{t}),
\end{aligned} \tag{B.2}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 \end{bmatrix}, \boldsymbol{\beta}_t = \begin{bmatrix} \boldsymbol{\beta}_{1,t} \\ f_{2,t} \end{bmatrix}, \mathbf{R}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{2,t} \end{bmatrix}, \boldsymbol{\alpha}_{\mathbb{S}_t} = \begin{bmatrix} \boldsymbol{\alpha}_{1,S_{1,t}} \\ \alpha_{2,S_{2,t}} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{F}_{1,2} \\ \mathbf{F}_{2,1} & \phi_{2,2} \end{bmatrix} \boldsymbol{\Omega}_\mathbf{t} = \begin{bmatrix} \boldsymbol{\Omega}_{1,t} & \boldsymbol{\Omega}_{1,2} \\ \boldsymbol{\Omega}_{2,1} & \sigma^2_{f_2} \end{bmatrix}.$$

More specifically,

$$\mathbf{H}_1 = \begin{bmatrix} \lambda_{1,1} & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \lambda_{2,1} & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots \\ \lambda_{8,1} & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & 0 \end{bmatrix} \mathbf{H}_2 = \begin{bmatrix} \lambda_{9,2} \\ \lambda_{10,2} \\ \vdots \\ \lambda_{19,2} \end{bmatrix} \boldsymbol{\beta}_{1,t} = \begin{bmatrix} f_{1,t} \\ \varepsilon_{1,t} \\ \varepsilon_{1,t-1} \\ \varepsilon_{1,t-2} \\ \varepsilon_{2,t} \\ \varepsilon_{2,t-1} \\ \varepsilon_{2,t-2} \\ \vdots \\ \varepsilon_{8,t} \\ \varepsilon_{8,t-1} \\ \varepsilon_{8,t-2} \end{bmatrix} \mathbf{R}_{2,t} = \begin{bmatrix} \sigma^2_{9,t}/\xi_t & 0 & \ldots & 0 \\ 0 & \sigma^2_{10,t}/\xi_t & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma^2_{19,t}/\xi_t \end{bmatrix}$$

$$
\boldsymbol{\alpha}_{1,S_{1,t}} = \begin{bmatrix} \alpha_{1,S_{1,t}} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{F}_1 = \begin{bmatrix} \phi_{1,1} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \psi_{1,1} & \psi_{1,2} & \psi_{1,3} & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \psi_{8,1} & \psi_{8,2} & \psi_{8,3} \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix}
$$

$$
\mathbf{Q}_{1,t} = \begin{bmatrix} \sigma^2_{f_1} & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{1,t}/\xi_t & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \sigma^2_{8,t}/\xi_t & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \end{bmatrix}
$$

The contemporaneous and temporal link between CEI and FCI in **linear** form is through the specifications of the $\mathbf{\Omega}_{1,2}$ and $\mathbf{F}_{1,2}, \mathbf{F}_{2,1}$ respectively. As we model the **nonlinear** link between CEI and FCI through their relation between the cyclical components, we set these matrices to zero to improve identification. Bayes factors computed using mildly informative priors favors these restrictions as well.

Appendix C Conditional Posterior Distributions

In this appendix, we derive the posterior distributions used in the sampling scheme described in Section 3.3 as follows

1. Sample f^T from $p(f^T|y^T, \alpha^{(m-1)}, \Phi^{(m-1)}, \Sigma^{(m-1)}, \mathbb{S}^{T(m-1)})$
2. Sample $\mathbb{S}^T$ from $p(\mathbb{S}^T|f^{T(m)}, \alpha^{(m-1)}, \Phi^{(m-1)}, \Sigma^{(m-1)}, \kappa^{(m-1)})$
3. Sample α from $f(\alpha|y^T, \mathbb{S}^{T(m)}, \Phi^{(m-1)}, \Sigma^{(m-1)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
4. Sample Φ from $f(\Phi|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \Sigma^{(m-1)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
5. Sample Σ from $f(\Sigma|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \Phi^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
6. Sample κ from $f(\kappa|y^T, S_1^{(m)}, \alpha^{(m)}, \Phi^{(m)}, \Sigma^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
7. Sample λ from $f(\lambda|y^T, f^{T(m)}, \sigma^{2(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
8. Sample σ^2 from $f(\sigma^2|y^T, f^{T(m)}, \lambda^{(m)}, \psi^{(m-1)}, \tau^{(m-1)})$
9. Sample ψ from $f(\psi|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \tau^{(m-1)})$
10. Sample τ from $f(\tau|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \psi^{(m)})$
11. Sample P from $f(P|S_1^{(m)})$
12. Repeat (1)-(11) M times.

C.1 Sampling of f_t Conditional on the discrete regimes and model parameters, the system (B.2) is a linear Gaussian state-space model and therefore, standard inference of the model can be carried out. This involves first running the Kalman filter forwards and running the simulation smoother backwards. The Kalman filter prediction steps are given in (13) in the main text. The remaining part of the Kalman filter is the updating steps, given as:

$$\begin{aligned}\boldsymbol{\beta}_{t|t} &= \boldsymbol{\beta}_{t|t-1} + \mathbf{K}_t \mathbf{v}_{t|t-1} \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{K}_t \mathbf{H}^* \mathbf{P}_{t|t-1}\end{aligned} \tag{C.1}$$

where $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}^{*'}\mathbf{V}_{t|t-1}^{-1}$ is the Kalman Gain. Once the Kalman filter is run forward, we can run a simulation smoother using the filtered values for drawing smoothed states as in Carter *et al.* (1994) and Frühwirth-Schnatter (1994). As this has become a standard practice in many applications, here we do not provide a detailed analysis but refer to standard textbooks such as Durbin and Koopman (2012).

C.2 Sampling of S_1^T To sample the discrete regime we employ a single-move sampler using the posterior density of $S_{1,t}$ as

$$f(S_{1,t}|S_1^{-t}, f^T, \theta) \propto f(S_{1,t}|S_{1,t-1}, \theta) f(S_{1,t+1}|S_{1,t}, \theta) \prod_{s=t-\kappa_{\min}}^{t+1+\kappa_{\max}} f(f_s|f^{s-1}, \mathbb{S}^s, \theta) \tag{C.2}$$

due to the Markov structure where $\kappa_{\max} = \max\{\kappa_0, \kappa_1\}$, $\kappa_{\min} = \min\{\kappa_0, \kappa_1\}$ and $X^t = \{X^1, \ldots, X^t\}$, $X^{-t} = \{X^1, \ldots, X^{t-1}, X^{t+1}, \ldots, X^T\}$.

Conditional on the factors, $f(f_s|f^{s-1}, \mathbb{S}^s, \theta)$ follows a Gaussian distribution derived from the standard regression framework with Gaussian error terms. The term $f(S_{1,t+1}|S_{1,t}, \boldsymbol{\theta})$ drops out at $t = T$. For $t = 1$, the term $S_{1,1}$ can be sampled from

$$f(S_{1,1}|S_1^{-1}, f^T, \theta) \propto f(S_{1,1}|\theta) f(S_{1,2}|S_{1,1}, \theta) \prod_{s=\max(0,1-\kappa_{\min})}^{2+\kappa_{\max}} f(f_s|f^{s-1}, \mathbb{S}^s, \theta) \tag{C.3}$$

where the unconditional density $f(S_{1,1}|\boldsymbol{\theta})$ follows a binomial density with probability $(1-p_1)/(2-p_1-q_1)$ derived from the ergodic probabilities of the Markov chain. Sampling of the state variables can be implemented by starting from the most recent value of S_1^T and sampling the states backward in time, one after another. After each step, the t^{th} element of S_1^T is replaced by its most recent draw.

We proceed with the estimation of the parameters that are related to the evolution of the common factors. For these parameters, we set up Metropolis Hastings samplers with candidates derived using the transition equations. The autoregressive process for the factors can be written as

$$f_{l,t} = (1 - S_{l,t})\alpha_{l,0} + S_{l,t}\alpha_{l,1} + \phi_{l,l} f_{l,t-1} + \eta_{l,t} \quad \eta_{l,t} \sim N(0, \sigma_{f_l}^2) \text{ for } l = 1, 2 \tag{C.4}$$

C.3 Sampling of $\boldsymbol{\alpha}_l$ for $l = 1, 2$ We use a Metropolis Hastings (MH) step to sample $\boldsymbol{\alpha}_l = (\alpha_{l,0}, \alpha_{l,1})'$ conditional on the data. For obtaining an efficient candidate density, we first restructure (C.4) as

$$\sigma_{f_l}^{-1}(f_{l,t} - \phi_{l,l} f_{l,t-1}) = \sigma_{f_l}^{-1}((1 - S_{l,t})\alpha_{l,0} + S_{l,t}\alpha_{l,1}) + \sigma_{f_l}^{-1}\eta_{l,t} \text{ for } l = 1, 2 \tag{C.5}$$

to form a regression as

$$Y_t = X_t \boldsymbol{\alpha}_l + v_{l,t} \qquad v_{l,t} \sim N(0,1)$$

To sample $\boldsymbol{\alpha}_l = (\alpha_{l,0}, \alpha_{l,1})'$ from the candidate density, we use a multivariate normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_2, \ldots, Y_T)'$ and $X = (X_2', \ldots, X_T')'$. As discussed in Section 3.2. in the main text, we impose restrictions on the elements of α_1 by sampling the parameters from the corresponding truncated distribution as the candidate density for identification of regimes. We then evaluate the probabilities conditional on the data, required to compute acceptance probability, using the Kalman filter given the draw from the candidate density.

C.4 Sampling of $\phi_{l,l}$ and $\sigma^2_{f_l}$ for $l = 1, 2$ In order to impose unit unconditional variance for the identification of the factors, we sample $\phi_{l,l}$ and σ^2_l jointly using the fact that $\sigma^2_{f_l} = (1 - \phi^2_{l,l})$ in case of unit unconditional variance. We use a MH step to sample $\phi_{l,l}$ and σ^2_l jointly. As in the previous case, for obtaining an efficient candidate density, we first restructure (C.4) as

$$\sigma^{-1}_{f_l}(f_{l,t} - \alpha_{l,S_{l,t}}) = \sigma^{-1}_{f_l} f_{l,t-1}\phi_{l,l} + \sigma^{-1}_{f_l}\eta_{l,t} \tag{C.6}$$

to form a regression as

$$Y_t = X_t \phi_{l,l} + v_{l,t} \qquad v_{l,t} \sim N(0,1)$$

To sample $\phi_{l,l}$ and σ^2_l from the candidate density, we use an multivariate normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_2, \ldots, Y_T)'$ and $X = (X_2', \ldots, X_T')'$. Stationarity is imposed by sampling the $\phi_{l,l}$ from the truncated distribution ensuring that $\phi_{l,l} < 1$. We optimize the density w.r.t. to this parameter using the restriction that $\sigma^2_{f_l} = (1 - \phi^2_{l,l})$ conditional on the factors to obtain a candidate draw for $\phi_{l,l}$ and therefore for $\sigma^2_{f_l}$. We then evaluate the probabilities conditional on the data, required to compute acceptance probability, using the Kalman filter given the draw from the candidate density.

C.5 Sampling of lead parameters $\boldsymbol{\kappa}$ As κ_0 and κ_1 parameters can only take discrete values we can compute the posterior probabilities for all $\boldsymbol{\kappa} \in \mathcal{C}$, where $\mathcal{C}$ defines restrictions and types of synchronization. We sample $\boldsymbol{\kappa}$ from the multinomial distribution, with the sampling occurring for both (κ_0, κ_1) parameters conditional on data rather than factors using a MH step. We can minimize the computational cost by using only the part that is related to the financial cycle S_2, as the shifts in S_1 and thus distinct values of $\boldsymbol{\kappa}$ are reflected as distinct values of S_2 while S_1 remains unaltered. Therefore, we decompose the Kalman filter recursion and the simulation smoother into parts for obtaining the kernel distribution $\boldsymbol{\kappa}$ which reduces the computational cost substantially.

Next, we proceed with parameters that are related to the measurement equation, which is rewritten below,

$$\begin{aligned} y_{i,t} &= \lambda_i f_t + \varepsilon_{i,t} \\ \psi(L)\varepsilon_{i,t} &= \epsilon_{i,t} \quad \epsilon_{it}|\xi_{i,t} \sim N(0, \sigma^2_{i,t}/\xi_{i,t}) \quad \xi_{i,t} \sim \Gamma(\tfrac{\nu}{2}, \tfrac{\nu}{2}) \\ \sigma^2_{i,t} &= \sigma^2_{i,1}\, \mathbb{I}[t \leq \tau] + \sigma^2_{i,2}\, \mathbb{I}[t > \tau] \ \text{ for } i = 1, \ldots, N. \end{aligned} \tag{C.7}$$

We first sample ξ_t using Gamma distribution update as

$$f(\xi_{i,t}|y_{i,t}, f_t, \sigma^2_{i,1}, \sigma^2_{i,2}, \psi_i(L), \lambda_i) \sim \begin{cases} \Gamma(\frac{v+1}{2}, \frac{v+\sigma^{-2}_{i,1}\psi_i(L)(y_{i,t}-\lambda_i f_t)^2}{2}) & \text{for} \quad t < \tau \\ \Gamma(\frac{v+1}{2}, \frac{v+\sigma^{-2}_{i,2}\psi_i(L)(y_{i,t}-\lambda_i f_t)^2}{2}) & \text{for} \quad t \geq \tau \end{cases} \tag{C.8}$$

see for example Albert and Chib (1993), to transform the system to follow a Gaussian distribution. Let $a_{i,t} \equiv \xi^{1/2}_{i,t}\epsilon_{i,t}$ and $e_{i,t} \equiv \xi^{1/2}_{i,t}\varepsilon_{i,t}$ denote the scaled error terms that follow Gaussian distributions.

C.6 Sampling of λ_i To sample λ_i we first transform the measurement equation by pre-multiplying with $\psi_i(L)$, $\xi_{i,t}$ and $\sigma^{-1}_{i,t}$ as

$$\sigma^{-1}_{i,t}\xi^{1/2}_{i,t}\Big(\psi_i(L)y_{i,t}\Big) = \sigma^{-1}_{i,t}\xi^{1/2}_{i,t}\Big(\psi_i(L)f_t\Big)\lambda_i + \sigma^{-1}_{i,t}\Big(\psi_i(L)e_{i,t}\Big) \tag{C.9}$$

for forming the following regression

$$Y_t = X_t\lambda_i + v_{i,t} \qquad v_{i,t} \sim N(0,1)$$

To sample λ_i, we use a normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_{k_i+1}, \ldots, Y_T)'$ and $X = (X'_{k_i+1}, \ldots, X'_T)'$. The lag structure of $\psi(L)$, k_i, is set as 3 for the economic variables whereas it is set to zero for the financial variables.

C.7 Sampling of $\sigma^2_{i,1}$ and $\sigma^2_{i,2}$ Following the transformation in the previous step we can sample $\sigma^2_{i,1}$ and $\sigma^2_{i,2}$ from an inverse-Gamma distributions with scale parameters $\left(\sum_{t=4}^{\tau-1} a^2_{i,t}\right)$ and $\left(\sum_{t=\tau}^{T} a^2_{i,t}\right)$ and degrees of freedom $(\tau - (k_i + 1))$ and $(T - \tau + 1)$, respectively.

C.8 Sampling of $\psi_i(L)$ To sample $\psi_i(L)$ we first transform the measurement equations by pre-multiplying it with $\sigma^{-1}_{i,t}$. For the regression equations regarding to economic variables with 3 lags of idiosyncratic factors, we can write

$$\sigma^{-1}_{i,t} e_{i,t} = \sigma^{-1}_{i,t} e_{i,t-1}\psi_{i,1} + \sigma^{-1}_{i,t} e_{i,t-2}\psi_{i,2} + e_{i,t-3}\psi_{i,3} + \sigma^{-1}_{i,t} a_{i,t} \tag{C.10}$$

to form a regression as

$$Y_t = X_t\Psi_i + v_{i,t} \qquad v_{i,t} \sim N(0,1)$$

where $\Psi_i = (\psi_{i,1}, \psi_{i,2}, \psi_{i,3})'$. To sample Ψ_i, we use a normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_{k_i+1}, \ldots, Y_T)'$ and $X = (X'_{k_i+1}, \ldots, X'_T)'$.

C.9 Sampling of τ The conditional posterior density of τ is as follows:

$$\begin{aligned} f(\tau|y^T, f^T, \theta) \propto \mathbb{I}[b + 4 \leq \tau \leq T - b] \times \prod_{i=1}^{N} \left((\sigma^{-1}_{i,1})^{(\tau-3)} (\sigma^{-1}_{i,2})^{(T-\tau+2)} \right) \times \\ \exp\left(-\frac{1}{2} \sum_{i=1}^{\hat{n}_1} \left(\sigma^{-2}_{i,1} \sum_{t=4}^{\tau-1} a^2_{i,t} + \sigma^{-2}_{i,2} \sum_{t=\tau}^{T} a^2_{i,t} \right) \right) \end{aligned} \tag{C.11}$$

where N is the number of variables. We can sample τ as discrete values from the range $[b + 4 \leq \tau \leq T - b]$ where $b = 12$ denoting the first and last 12 observations.

C.10 Sampling of p_i and q_i The conditional posterior densities of the transition parameters are given by

$$\begin{aligned} f(p_i \mid S_i) &\propto p_i^{T_{00}+N_{00}-1}(1-p_i)^{T_{01}+N_{01}-1} \\ f(q_i \mid S_i) &\propto q_i^{T_{10}+N_{10}-1}(1-q_i)^{T_{11}+N_{11}-1} \end{aligned} \tag{C.12}$$

where T_{ij} denotes the number of transitions from state i to state j and N_{ij} denotes the corresponding parameters regarding to prior distribution. This corresponds to the kernel of a Beta distribution. Therefore, the transition probabilities can be sampled from a Beta distribution with parameters $T_{ij} + N_{ij}$.

Appendix D Estimation results of the competing models

Table D.5: Estimates of factor loadings

		Imperfect synchronization of cycles	Perfect synchronization of cycles	Independent cycles
Economic variables				
ip	$\lambda_{1,1}$	0.434 (0.079)	0.418 (0.079)	0.401 (0.092)
import	$\lambda_{2,1}$	0.259 (0.068)	0.253 (0.067)	0.246 (0.081)
export	$\lambda_{3,1}$	0.115 (0.055)	0.109 (0.053)	0.097 (0.054)
retails	$\lambda_{4,1}$	0.405 (0.112)	0.383 (0.114)	0.361 (0.137)
pmi	$\lambda_{5,1}$	0.169 (0.151)	0.177 (0.153)	0.187 (0.158)
empna	$\lambda_{6,1}$	0.113 (0.117)	0.136 (0.119)	0.141 (0.126)
traserv^q	$\lambda_{7,1}$	0.236 (0.154)	0.252 (0.157)	0.241 (0.155)
traserv^m	$\lambda_{8,1}$	0.419 (0.112)	0.397 (0.115)	0.364 (0.134)
Financial variables				
rbist	$\lambda_{9,2}$	0.576 (0.066)	0.577 (0.066)	0.575 (0.067)
FXRes	$\lambda_{10,2}$	0.260 (0.070)	0.262 (0.071)	0.261 (0.071)
Conf	$\lambda_{11,2}$	0.607 (0.072)	0.612 (0.071)	0.606 (0.072)
TermS	$\lambda_{12,2}$	0.290 (0.084)	0.293 (0.084)	0.292 (0.085)
VOL	$\lambda_{13,2}$	-0.238 (0.078)	-0.239 (0.078)	-0.238 (0.078)
P/E	$\lambda_{14,2}$	0.184 (0.104)	0.184 (0.103)	0.186 (0.104)
TAuc	$\lambda_{15,2}$	-0.311 (0.075)	-0.311 (0.076)	-0.311 (0.075)
TETS	$\lambda_{16,2}$	-0.117 (0.059)	-0.118 (0.065)	-0.117 (0.063)
Cred	$\lambda_{17,2}$	-0.180 (0.095)	-0.180 (0.096)	-0.184 (0.096)
MSCIem	$\lambda_{18,2}$	0.643 (0.095)	0.645 (0.095)	0.640 (0.096)
EMBI-Tr	$\lambda_{19,2}$	0.104 (0.038)	0.105 (0.042)	0.105 (0.041)

Note: The table shows posterior means and standard deviations (in parentheses) of the factor loading parameters in the measurement equations in (11) in the main text for the competing models estimated using the data for the periods starting from January 1999 until October 2018. The competing models are constituted by the model with Imperfectly Synchronized phase synchronized with regime dependent phase shifts between the cyclical components of the CEI and the FCI, the model with Perfectly Synchronized cycles (PS) for the CEI and FCI and the model with independent cycles for the CEI and FCI. Posterior results are based on 60,000 draws from the posterior distribution where the first 10,000 draws are discarded as burn-in sample.

Table D.6: Estimates of conditional variances of the variables

		Imperfect synchronization of cycles	Perfect synchronization of cycles	Independent cycles
Most likely break date	τ	2001 : 09	2001 : 09	2001 : 09
Economic variables				
ip	$\sigma^2_{1,1}$	1.095 (0.315)	1.105 (0.319)	1.112 (0.321)
	$\sigma^2_{1,2}$	0.731 (0.106)	0.739 (0.106)	0.751 (0.110)
import	$\sigma^2_{2,1}$	1.987 (0.633)	1.987 (0.636)	1.995 (0.642)
	$\sigma^2_{2,2}$	0.631 (0.097)	0.633 (0.101)	0.632 (0.100)
export	$\sigma^2_{3,1}$	1.263(0.389)	1.268(0.391)	1.263(0.393)
	$\sigma^2_{3,2}$	0.592(0.071)	0.591(0.071)	0.596(0.072)
retails	$\sigma^2_{4,1}$	1.622(2.034)	1.630(2.670)	1.611(2.088)
	$\sigma^2_{4,2}$	0.775(0.149)	0.794(0.155)	0.805(0.158)
pmi	$\sigma^2_{5,1}$	1.648(2.103)	1.645(2.153)	1.635(2.419)
	$\sigma^2_{5,2}$	0.933(0.160)	0.932(0.163)	0.926(0.161)
empna	$\sigma^2_{6,1}$	1.615(2.074)	1.612(2.137)	1.614(2.228)
	$\sigma^2_{6,2}$	0.889(0.114)	0.882(0.114)	0.880(0.114)
traserv^q	$\sigma^2_{7,1}$	1.620(2.042)	1.617(3.859)	1.642(2.594)
	$\sigma^2_{7,2}$	0.921(0.229)	0.912(0.239)	0.916(0.235)
traserm^m	$\sigma^2_{8,1}$	1.600(2.040)	1.611 (2.228)	1.611 (2.031)
	$\sigma^2_{8,2}$	0.736 (0.132)	0.754 (0.135)	0.777 (0.139)
Financial Variables				
rbist	$\sigma^2_{9,1}$	1.877(0.618)	1.877(0.619)	1.871(0.622)
	$\sigma^2_{9,2}$	0.352(0.069)	0.352(0.069)	0.351(0.071)
FXRes	$\sigma^2_{10,1}$	3.325(1.138)	3.320(1.148)	3.318(1.147)
	$\sigma^2_{10,2}$	0.510(0.072)	0.510(0.074)	0.509(0.072)
Conf	$\sigma^2_{11,1}$	0.635(0.217)	0.621(0.213)	0.639(0.218)
	$\sigma^2_{11,2}$	0.627(0.091)	0.626(0.091)	0.628(0.091)
TermS	$\sigma^2_{12,1}$	1.735(2.976)	1.712(2.083)	1.709(1.868)
	$\sigma^2_{12,2}$	0.721(0.129)	0.722(0.132)	0.719(0.132)
VOL	$\sigma^2_{13,1}$	1.266(0.333)	1.264(0.330)	1.263(0.330)
	$\sigma^2_{13,2}$	0.899(0.122)	0.898(0.123)	0.897(0.123)
P-E	$\sigma^2_{14,1}$	2.190(1.252)	2.188(1.260)	2.202(1.266)
	$\sigma^2_{14,2}$	0.681(0.318)	0.682(0.318)	0.676(0.321)
TAuc	$\sigma^2_{15,1}$	1.622(0.472)	1.614(0.472)	1.622(0.475)
	$\sigma^2_{15,2}$	0.776(0.111)	0.776(0.111)	0.773(0.111)
TETS	$\sigma^2_{16,1}$	10.441(5.965)	10.411(5.949)	10.332(5.903)
	$\sigma^2_{16,2}$	0.083(0.027)	0.083(0.028)	0.083(0.028)
Cred	$\sigma^2_{17,1}$	1.589(2.135)	1.594(2.016)	1.596(2.323)
	$\sigma^2_{17,2}$	0.893(0.206)	0.896(0.219)	0.895(0.212)
MSCIem	$\sigma^2_{18,1}$	1.587(2.014)	1.598(2.072)	1.570(2.063)
	$\sigma^2_{18,2}$	0.651(0.108)	0.650(0.108)	0.653(0.110)
EMBI-Tr	$\sigma^2_{19,1}$	6.624(3.782)	6.606(3.776)	6.581(3.793)
	$\sigma^2_{19,2}$	0.055(0.024)	0.055(0.024)	0.055(0.024)

Note: The table shows posterior means and standard deviations (in parentheses) of the variances of the idiosyncratic components in the measurement equations in (11) in the main text for the competing models estimated using the data for the periods starting from January 1999 until October 2018. Posterior results are based on 60,000 draws from the posterior distribution where the first 10,000 draws are discarded as burn-in sample. See Table D.5 for further details.

Table D.7: Autoregressive coefficients of the idiosyncratic factors of economic variables

		Imperfect synchronization of cycles	Perfect synchronization of cycles	Independent cycles
	$\psi_{1,1}$	-0.230 (0.085)	-0.242 (0.085)	-0.244 (0.084)
ip	$\psi_{1,2}$	-0.070 (0.083)	-0.078 (0.082)	-0.079 (0.083)
	$\psi_{1,3}$	0.003 (0.079)	-0.002 (0.078)	-0.003 (0.078)
	$\psi_{2,1}$	-0.394 (0.078)	-0.397 (0.079)	-0.401 (0.078)
import	$\psi_{2,2}$	-0.059 (0.084)	-0.062 (0.084)	-0.063 (0.084)
	$\psi_{2,3}$	0.054 (0.076)	0.052 (0.077)	0.051 (0.076)
	$\psi_{3,1}$	-0.582 (0.069)	-0.583 (0.068)	-0.582 (0.068)
export	$\psi_{3,2}$	-0.314 (0.076)	-0.316 (0.076)	-0.314 (0.076)
	$\psi_{3,3}$	-0.073 (0.067)	-0.076 (0.066)	-0.075 (0.066)
	$\psi_{4,1}$	-0.358 (0.131)	-0.369 (0.129)	-0.374 (0.128)
retails	$\psi_{4,2}$	-0.117 (0.136)	-0.125 (0.136)	-0.130 (0.136)
	$\psi_{4,3}$	-0.082 (0.125)	-0.083 (0.123)	-0.083 (0.123)
	$\psi_{5,1}$	-0.030 (0.116)	-0.029 (0.115)	-0.033 (0.116)
pmi	$\psi_{5,2}$	-0.167 (0.111)	-0.168 (0.111)	-0.171 (0.111)
	$\psi_{5,3}$	0.037 (0.114)	0.039 (0.114)	0.037 (0.115)
	$\psi_{6,1}$	0.128 (0.085)	0.123 (0.085)	0.122 (0.084)
empna	$\psi_{6,2}$	0.275 (0.079)	0.273 (0.079)	0.272 (0.079)
	$\psi_{6,3}$	-0.183 (0.080)	-0.183 (0.081)	-0.184 (0.081)
	$\psi_{7,1}$	0.011 (0.168)	-0.003 (0.171)	0.002 (0.169)
traserv^q	$\psi_{7,2}$	0.135 (0.159)	0.134 (0.160)	0.135 (0.159)
	$\psi_{7,3}$	0.149 (0.160)	0.150 (0.160)	0.147 (0.160)
	$\psi_{8,1}$	-0.307 (0.119)	-0.325 (0.117)	-0.324 (0.114)
traserv^m	$\psi_{8,2}$	-0.078 (0.119)	-0.088 (0.119)	-0.083 (0.118)
	$\psi_{8,3}$	0.113 (0.116)	0.108 (0.115)	0.111 (0.114)

Note: The table shows posterior means and standard deviations (in parentheses) of the autoregressive coefficients of the idiosyncratic factors of economic variables in the measurement equations in (11) in the main text for the competing models estimated using the data for the periods starting from January 1999 until October 2018. Posterior results are based on 60,000 draws from the posterior distribution where the first 10,000 draws are discarded as burn-in sample. See Table D.5 for further details.

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