PLASMA IONIZATION **BY** HELICON **WAVES**

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Introduction

- Helicon waves belong to whistler waves which are RHP EM waves in free space
- Helicon waves excitation used to make dense plasma source (Boswell - 1970)
- Low freq allows neglecting electrons gyration
- They are no more purely EM in bounded regions
- Landau damping explains absorption and ionization efficiency of helicon waves, also used to accelerate primary electrons(Chen-1985-1987)

Introduction

- An average density **of** $\sim 5 \times 10^{12}\ cm^{-3}$, with 1 kW of r.f. power, is an order of magnitude improvement over that in ordinary discharges and brings *W* down to the order of the ionization energy.

- We hypothesize that this is possible if the ionizing electrons are directly accelerated by the wave particle interaction rather than by a random heating process.

- This paper gives the theoretical basis for this hypothesis.

Dispersion Relation

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

$$\mathbf{E} = \mathbf{j} \times \mathbf{B}_0 / e n_0,$$

B along z gives

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{j} = 0$$

$$\mathbf{j}_\perp = -e n_0 \mathbf{E} \times \mathbf{B}_0 / B_0^2.$$

We neglected

1-Displacement current

2-Ion motion since we assume frequency much higher than lower hybrid frequency

3-resistivity so $E_z = 0$

We assumed the plasma current is entirely carried by the E x **B** guiding center drift of the electrons since $\omega \ll \omega_c$

Dispersion Relation

- We have

$$i\omega \mathbf{B} = \nabla \times \mathbf{E} = \nabla \times (\mathbf{j} \times \mathbf{B}_0)/en_0$$
$$= (\mathbf{B}_0 \cdot \nabla)\mathbf{j}/en_0 \approx (ikB_0/en_0)\mathbf{j},$$

- Substituting for J we get

$$\nabla \times \mathbf{B} = \alpha \mathbf{B},$$

- Where

$$\alpha \equiv \frac{\omega}{k} \frac{\mu_0 e n_0}{B_0} = \frac{\omega}{k} \frac{\omega_p^2}{\omega_c c^2}.$$

- We get

$$\mathbf{j} = (\alpha/\mu_0)\mathbf{B},$$

- Solving for B $_z$ we get a Bessel function (finite at r=0), Br and Bɵ are deduced

$$B_z = C_3 J_m(Tr) \quad B_r = \frac{iC_3}{T^2}\left(\frac{m}{r}\alpha J_m - kJ'_m\right) \quad B_\theta = \frac{C_3}{T^2}\left(\frac{m}{r}kJ_m + \alpha J'_m\right).$$

- where

$$T^2 \equiv \alpha^2 - k^2.$$

Dispersion Relation

For future reference, we give here the right- hand and left-hand circular components B_R, B_L of the local field as defined by

$$\sqrt{2}B_R \equiv B_r - iB_\theta, \quad \sqrt{2}B_L \equiv B_r + iB_\theta$$

The electric field **E is given by**

$$E_r = (\omega/k)B_\theta, \quad E_\theta = -(\omega/k)B_r, \quad E_z = 0,$$

For the case of the simplest helicon, it does not matter whether the tube is insulating or conducting $B_r=0$

since $J_r(a)=0$ 0r $E_\theta(a)=0$

Dispersion Relation

The boundary condition gives

$$m\alpha J_m(Ta) + kaJ'_m(Ta) = 0,$$

In particular, the lowest two azimuthal modes are given by

$$J_1(Ta) = 0 \qquad (m = 0)$$

$$J_1(Ta) = (kaT/2\alpha)(J_2 - J_0) \simeq 0 \quad (m = 1).$$

The last inequality holds for long, thin tubes, where

$$T \simeq \alpha \text{ and } ka \ll 1$$

Dispersion Relation

The resulting approximate dispersion relation for ***m > I can then be written as***

$$\frac{B_0}{n_0} = \frac{e\mu_0 a}{Z_m}\left(\frac{\omega}{k} + \frac{\omega a}{m Z_m^2}\right),$$

$$J_m(Z_m) = 0$$

The second term is a small correction of order *k/T and* is essentially an additive constant. We see that *B/n is proportional to the phase* velocity; that is, to the square root of the accelerated electron energy E_f*. Thus, if* E_f has an optimum value for efficient ionization, *the ratio* ***n/B tends to be constant.***

$$Z_m = 3.83 \text{ for } m = 1$$

Structure of Helicon Modes

- The nonzero large z component of B conserves its divergence less nature, while E_z is zero and its divergence is proportional to B_z

- The wave fields are given by

$$B_r = -\frac{k}{\omega} E_\theta = \frac{2A}{T}\left[\frac{m\alpha}{r} J_m(Tr) + kJ'_m(Tr)\right]\cos(m\theta + kz - \omega t)$$

$$B_\theta = \frac{k}{\omega} E_r = -\frac{2A}{T}\left[\alpha J'_m(Tr) + \frac{mk}{r} J_m(Tr)\right]\sin(m\theta + kz - \omega t)$$

- For m=0 mode,

$$B_r = -AkJ_1(Tr)\cos\psi, \quad E_r = A\omega(\alpha/k)J_1(Tr)\sin\psi$$

$$B_\theta = A\alpha J_1(Tr)\sin\psi, \quad E_\theta = A\omega J_1(Tr)\cos\psi$$

$$B_z = ATJ_0(Tr)\sin\psi, \quad E_z = 0,$$

m=0 mode

- Where $\psi = kz - \omega t.$

- When Ψ = 0, E vanishes, and the field is purely electromagnetic. When Ψ = ***π/4, the field is purely radial and electrostatic.*** In between, the field lines are spiral.
- Since *la/kl is normally >> 1, the radial electrostatic* component of **E** dominates over the azimuthal, electromagnetic component,

- This suggests that coupling to this mode is best done through the electrostatic field. The smaller |*k/a*| *is, the smaller the range of phase angles* Ψ *over which the electromagnetic* component of E can be seen; and in the limit *k/a = 0, the E-field is always radial (*space charge field), changing sign at Ψ= *n/2.*

m=0 mode

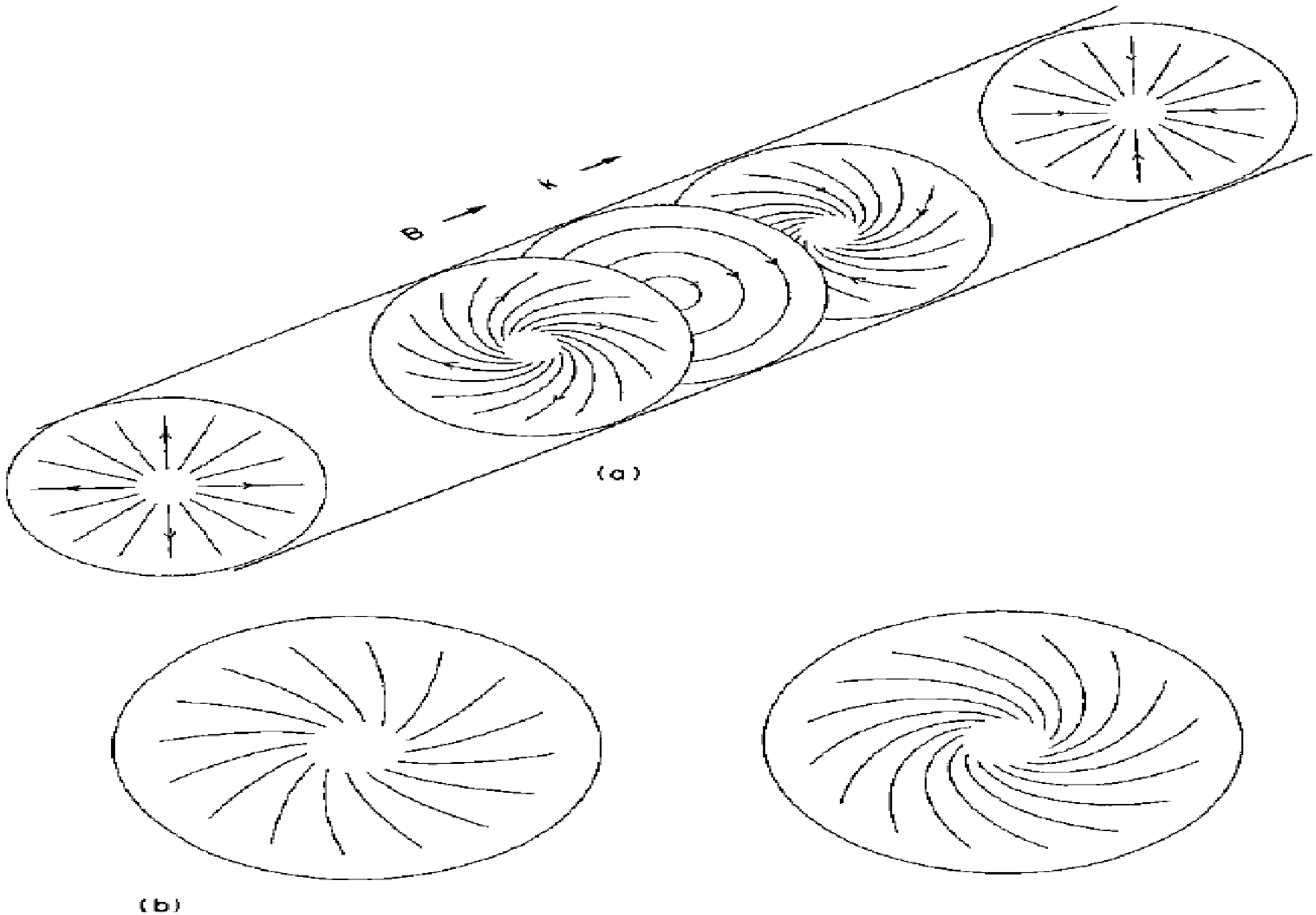

FIG. 1.—Electric field line patterns for the $m = 0$ mode. (a) 3-D representation; (b) cross-sections at $(k/\alpha)\,\mathrm{ctn}\,(kz - \omega t) = 1/3$ (left) and 1 (right).

m=+1,-1 mode

By contrast, the $m = +1$ mode has a field pattern that does not change with position; but it does change with the value of $|k/\alpha|$. The electric field pattern is given by

$$E_r = -\frac{A}{T}\frac{\omega}{k}\left[\alpha J'_1(Tr) + \frac{k}{r}J_1(Tr)\right]\sin(\theta + kz - \omega t) = \frac{\omega}{k}B_\theta$$

$$E_\theta = -\frac{A}{T}\frac{\omega}{k}\left[\frac{\alpha}{r}J_1(Tr) + kJ'_1(Tr)\right]\cos(\theta + kz - \omega t) = -\frac{\omega}{k}B_r,$$

Near the axis, the m = 1 mode is right-hand polarized while at the boundary, it is plane polarized since E must be perpendicular to the boundary*. In between, there* is a region in which *E is left-hand elliptically polarized.* The transverse components of B induce an electromagnetic *E***,** which cancels the *E: caused by the space charge; in this way, the total E: is made* zero, as it has to be in the absence of damping. In the limit *k/α = 0, the* pattern becomes the same as that of the TM_{11} *electromagnetic mode in a vacuum* waveguide.

m=+1,-1 mode

The pattern simply rotates as z changes to keep ɵ+kz cst

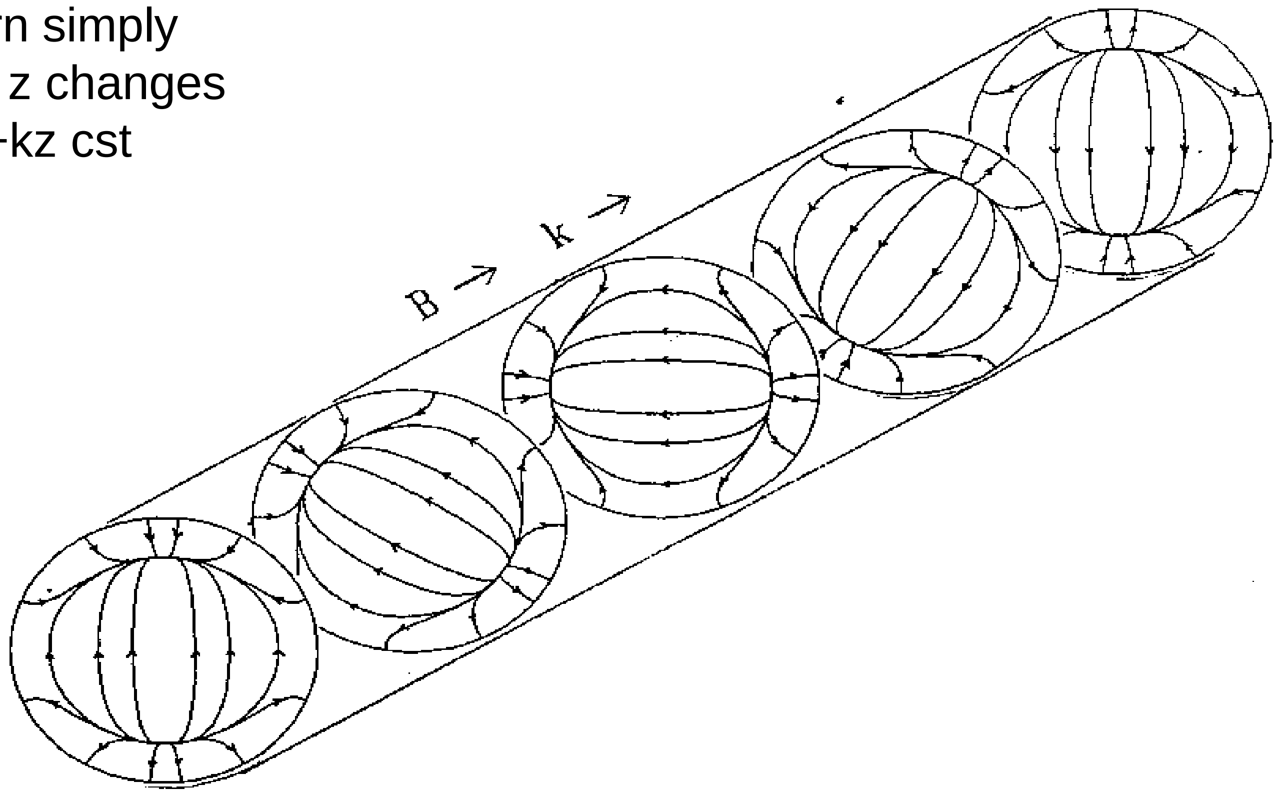

Figure 3. Instantaneous electric field pattern for an m = +1 helicon wave in space (Ref. 27).

m=+1,-1 mode

- An m=1 antenna can be designed to couple the strong electrostatic E field at the center
- The complementary m= -1 mode allows when adding both modes to get a mode that is nearly plane-polarized everywhere, and thus susceptible to being driven by a non-helical antenna. A discussion of this interesting problem will be given in a separate paper by Chen.
- Indeed, the TE helicon mode resembles the TM electromagnetic mode. This shows the importance of the space charge field in helicon waves.

m=+1,-1 mode

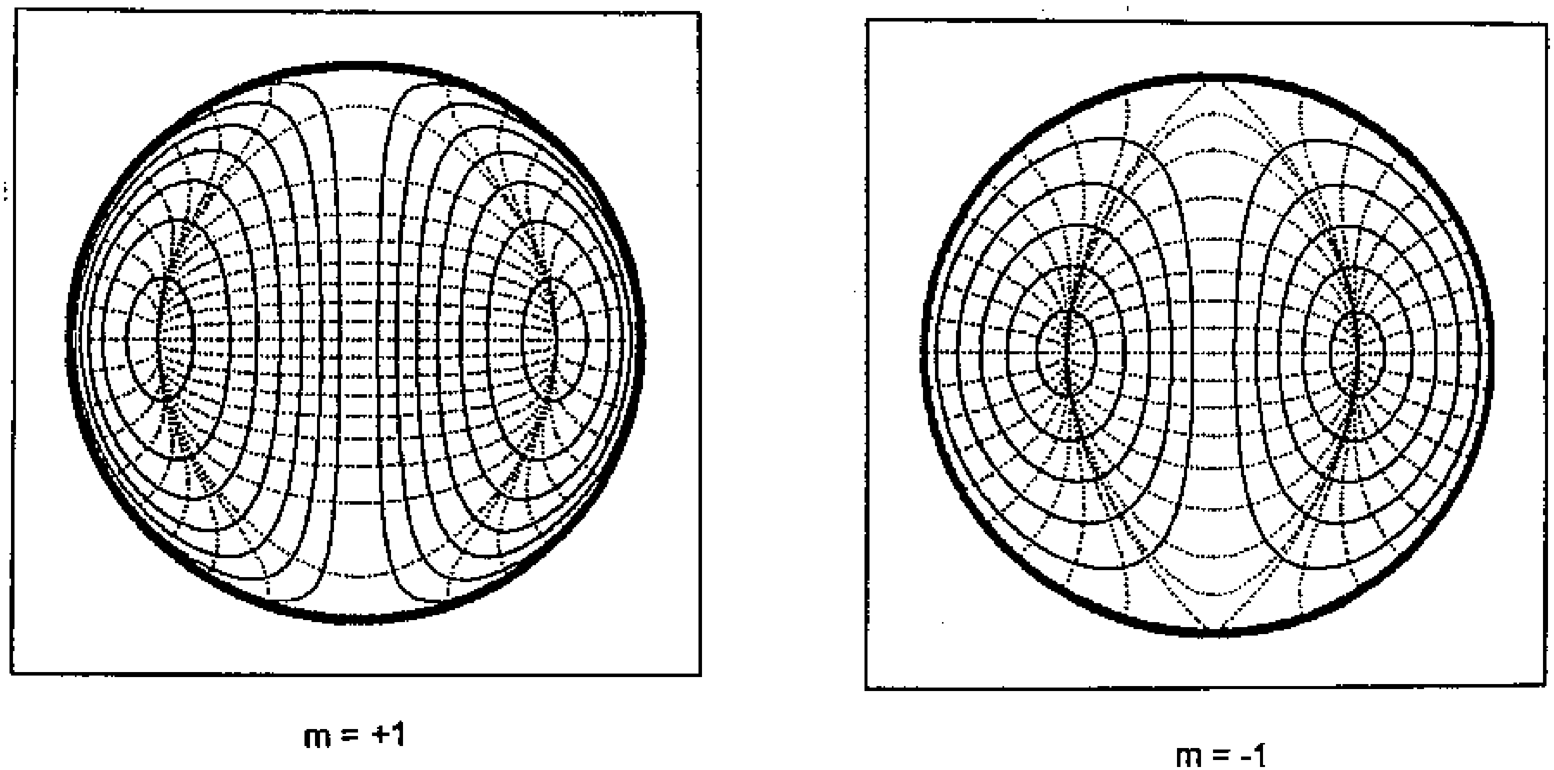

Figure 2. Pattern of magnetic (solid) and electric (dashed) field lines in the m = +1 and -1 modes of the helicon wave in a uniform plasma in a plane perpendicular to the dc magnetic field (Ref. 35).

m=+1,-1 mode

$$T^2 \equiv \alpha^2 - k^2.$$

the radius of maximum energy deposition **is given by**

$$J'_1(Tr_m) = 0.$$

All the field lines converge on a point at a radius r_o

$$\beta J_0(Tr_0) - J_2(Tr_0) = 0.$$

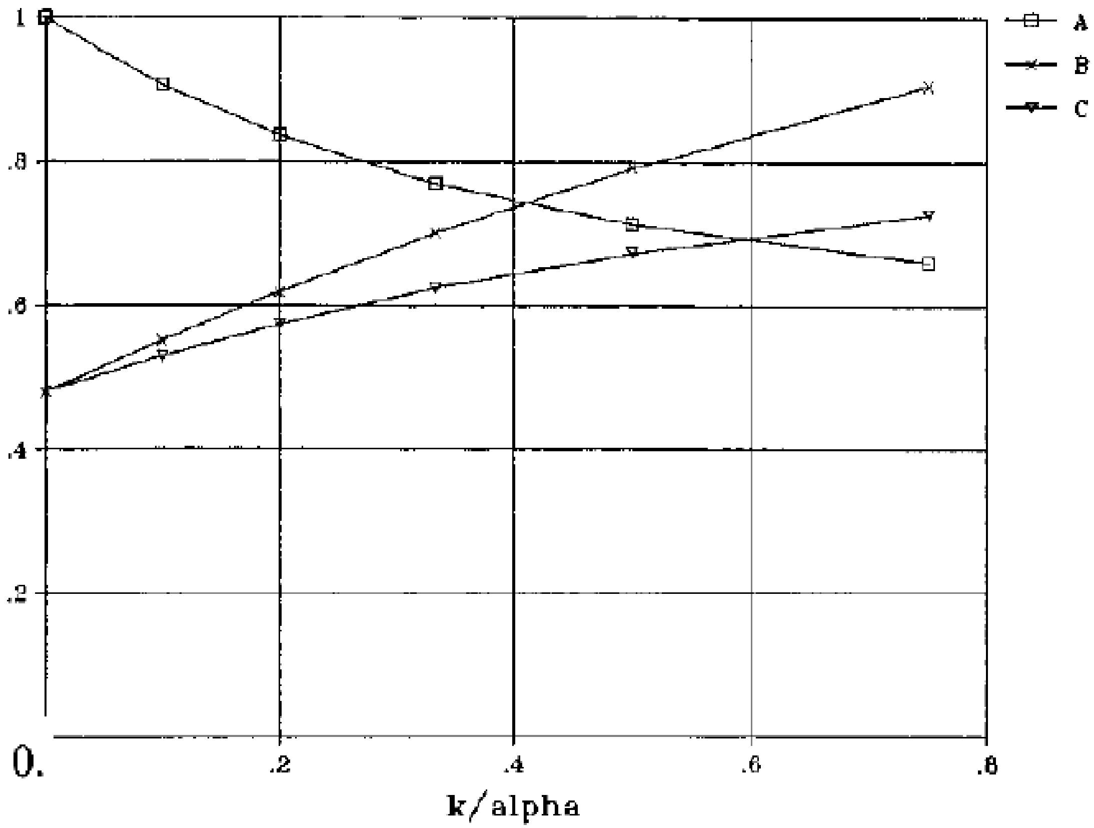

FIG. 3.—For the $m = 1$ mode, variation with k/α of (A) $T/3.83$, (B) r_0 and (C) r_m.

Collisional and Collisionless Damping

- Damping of helicon waves arises, as with AIfven waves, from the drag on electron motion along B caused by collisions or by Landau damping.
- A component E_z *is then* needed to push the electrons in that direction.
- To arrive at simple formulae for the damping, we assume the ordering

$$\nu \ll \omega \ll \omega_c$$

- Which is valid over a wide parameter regime.
- Electrons collision rate with neutrals is negligible with respect to that with ions.
- Electron inertia is dominant in the parallel motion, so we only need to modify J_z
- We shall treat in different paper fields below 100G, since electron gyratory motion and perpendicular motion should be considered.

Collisional Damping

The linearized equation of motion for a cold electron fluid with a phenomenological collision rate yields

$$\mathbf{E} = \frac{\mathbf{j} \times \mathbf{B}_0}{e n_0} - \frac{i m}{n_0 e^2} (\omega + i\nu)\mathbf{j}.$$

The solution for $B = B_1 + B_2$ gives

$$\nabla \times \mathbf{B}_1 = \beta_1 \mathbf{B}_1, \quad \nabla \times \mathbf{B}_2 = \beta_2 \mathbf{B}_2.$$

With

$$\beta_{1,2} = [1 \mp (1 - 4\alpha\gamma)^{1/2}]/2\gamma, \qquad \gamma \equiv (\omega + i\nu)/k\omega_c,$$

The root β_1 is that corresponding to the helicon wave, since $\beta_2 \simeq 1/2\gamma$ is not close to α. For small $\alpha\gamma$, β_1 is approximately

$$\beta_1 = \frac{1}{2\gamma} [1 - (1 - 4\alpha\gamma)^{1/2}] \cong \alpha(1 + \alpha\gamma).$$

Collisional Damping

Where $\gamma \equiv (\omega + \mathrm{i}\nu)/k\omega_c,$

When ν is the electron–ion collision frequency ν_{ei}, the plasma resistivity η is

$$\eta = m\nu/n_0 e^2,$$

Due to boundary conditions T is real so K must be complex

$$T^2 = \beta_1^2 - k^2$$

$$k = k_r + k_i, \quad \delta \equiv k_i/k_r$$

for $T \gg k$. The damping rate is then approximately

$$\mathrm{Im}\,(k) \simeq \alpha\nu/\omega_c \simeq (\nu/\omega_c)T.$$

Collisional Damping

The collisional damping length $L_c = 1/\mathrm{Im}(k)$ can be written as

$$L_c \simeq \frac{\omega_c}{\nu T} \simeq \frac{\omega}{k} \frac{\mu_0}{\eta T^2}.$$

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Landau Damping

In the absence of collisions, the electron motion along $\mathbf{B}_0$ is controlled by electron inertia and wave–particle interactions. The inertia effect was included in the previous section in the limit $\nu = 0$, but the Landau effect has to be treated kinetically.

The only modification is the use of Boltzmann equation to account for the parallel motion of the electrons

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial z} - \frac{e}{m}E\frac{\partial f_0}{\partial v} = \left(\frac{n_1}{n_0}f_0 - f_1\right)\nu.$$

$$\nu_{\mathrm{LD}} = 2\sqrt{\pi}\omega\zeta^3\,\mathrm{e}^{-\zeta^2}.$$

Landau Damping

The total effective collision frequency is then

$$\nu_{\rm eff} = \nu + \nu_{\rm LD}.$$

$$\nu_{\rm LD}(\max) \simeq 2\sqrt{\pi}(0.41)\omega = 1.45\omega.$$

the break-even density at which $\nu = \nu_{\rm LD}(\max)$ is

$$n_e = 2.6 \times 10^{11}\omega = 1.63 \times 10^{12} f.$$

Thus, Landau damping should be the dominant dissipation mechanism for densities below about 5×10^{19} m^{-3} for $f \simeq 30$ MHz.

Conclusion

- Helicon waves have shown efficiency in generating plasma.
- The efficiency of helicon waves is interpreted by the phenomenon of Landau damping.
- The dispersion relation is concluded by solving the wave equation and incorporating Maxwell's equations and the fluid equation of motion along with assumptions taken to simplify the calculation.
- The collision frequency is calculated for the case of collisional damping.
- In the case of Landau damping, the effective collision frequency is calculated by incorporating Boltzmann equation which accounts for the kinetic effects.
- It is shown that the Landau collision frequency is proportional to the frequency of the wave and attains a maximum at a break-even density.