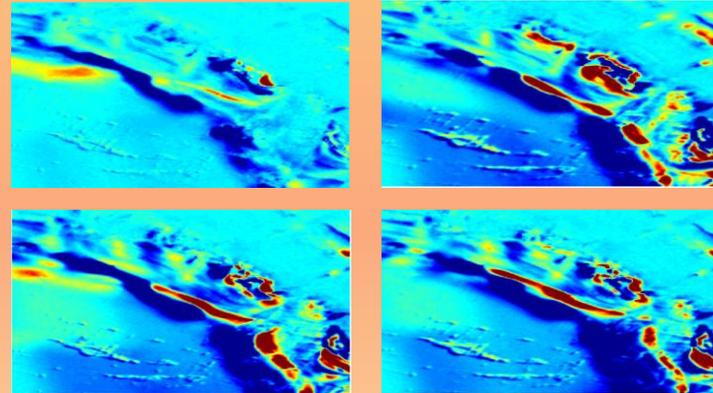




Presentation entitled

Fluid motion in a cylindrical container subject to electromagnetic forces

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OUTLINE



- Motivation and goal
- Experimental setup
- Assessing X-ray imaging diagnostic
- The flow dynamics by MHD dimensionless parameters .
- Experimental Results , with and without the oxidized layer .
- Conclusion
- Future work

Motivation and goal

Goal :

This research work is to study diagnostic methods aiming at characterizing the flow dynamics of a conducting liquid metal. It should also reveal the instability and the quasi-2D motion in the flow driven by electromagnetic forces.

Motivation:

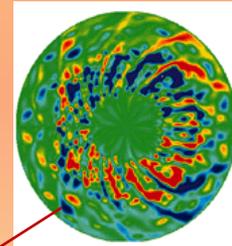
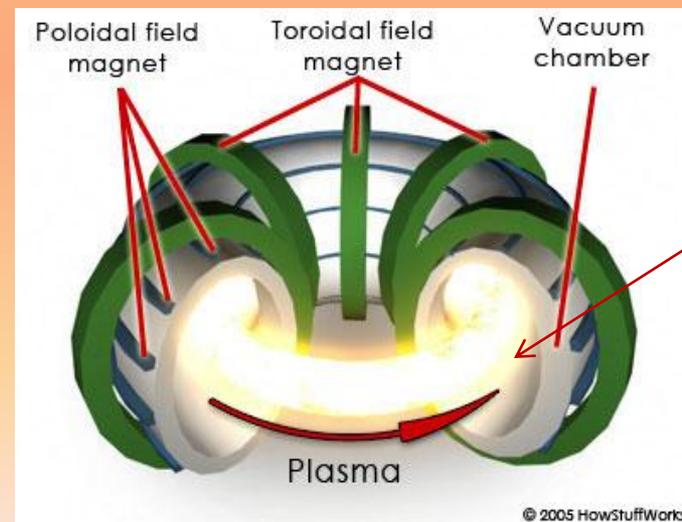
This experimental study is motivated by two research areas:

1- *Plasma physics:*

Studying turbulence of plasma in magnetic fusion devices.

Most of the turbulent motion happens

\perp to the toroidal magnetic field. Such turbulence is caused by *Kelvin-Helmoltz instability* due to ∇u or/and *Taylor* instabilities due to plasma $\nabla \rho$.





Geophysics:

In the atmosphere and even the oceans the vortex motion is 2D as long as the thickness layer of the fluid is small compared to the radius of the earth. But this thickness becomes an important factor when it increases developing a 3D motion out of the 2D .



2D von Karman Vortex street clouds

Experimental investigations urged us to make a table top experiment with a conducting liquid forced to circulate under Lorentz force .

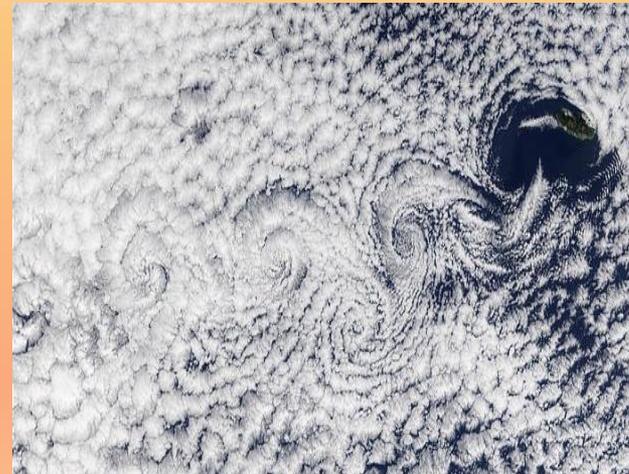
Geophysics:

In the atmosphere and the oceans the vortex motion may be considered 2D as long as the thickness layer of the fluid is small compared to the Earth radius.

This thickness is an important factor when it increases allowing turbulence to develop a 3D motion.



small bathtub tornado



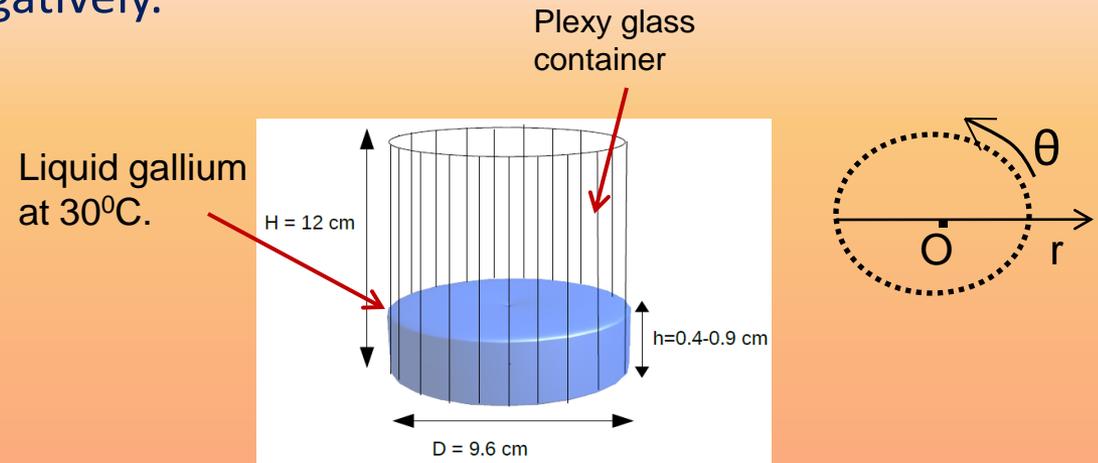
2D von Karman Vortex street clouds

To investigate experimentally the 2D vs. quasi-2D aspects, a table top experiment with a conducting liquid forced to circulate under the Lorentz force is designed and built.

The Experimental set-up

Electric current

- 36 stainless steel electrodes (12 cm each) installed . Could be alternately biased positively and negatively.



A steady current passes through the electrodes up to 10 Amperes is obtained by setting the potential difference across the electrodes and by choosing a certain number of electrodes.

It gives a current density of $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity of gallium.

The electric field strength $\mathbf{E} = E(r, \theta) = -\nabla \varphi(r, \theta)$, where $\varphi(r, \theta)$ is the potential inside the liquid gallium.

Liquid Gallium Properties

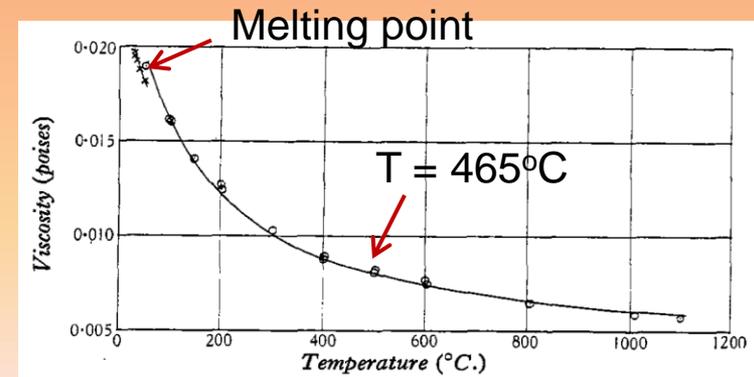
- It is silvery-like metal which is liquid at room temperature (29.8°C).
- It has many physical properties that make it a good candidate in magnetohydrodynamic (MHD) studies.
- Other physical properties at 30°C :
 - High conductivity: $\sigma = 3.85 \times 10^6 \Omega\cdot\text{m}$
 - Low viscosity $\nu = 3.22 \times 10^{-7} \text{N}\cdot\text{m}^{-2}\cdot\text{s}$
 - It has a high surface tension which depends on temperature (T):



$$\gamma = 708 - 0.66(T - 29.8)$$

→ $\gamma = 708 \text{ mN}$ at the melting point .

- It reacts with oxygen easily to form gallium oxide layer on the surface which is non-conducting at temperatures below 100°C .
- The kinematic viscosity at 465°C is $\nu = 1.5 \times 10^{-7} \text{m}^2/\text{s}$.





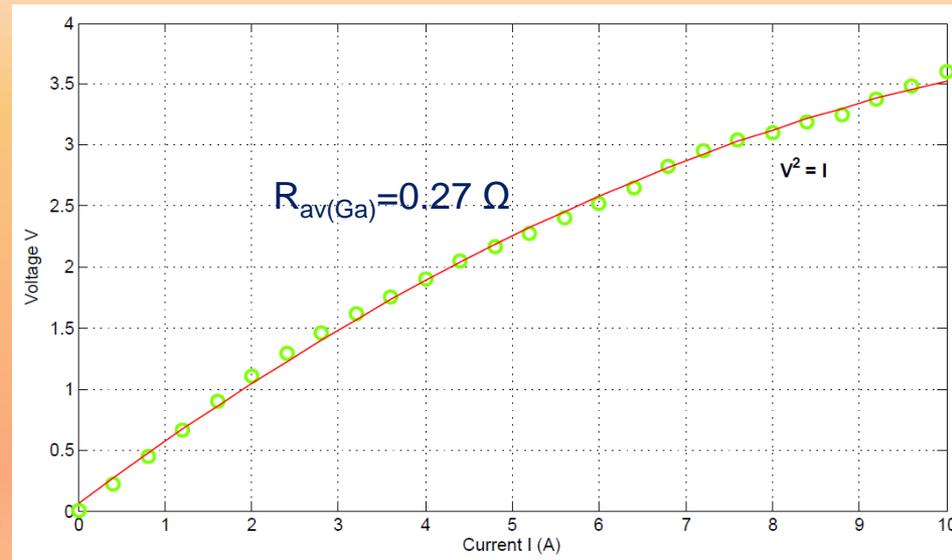
Resistance of Gallium

The measured I-V curve reveals the **semi-conductor** property of gallium. The cylindrical geometry of gallium yields a resistance with an expression

$$R_{Ga} = \frac{\rho_{Ga}}{2h} \sin^{-1}\left(\frac{x - R}{R}\right)$$

Where:

R is the radius of the container.
The gallium resistivity is $28 \times 10^{-6} \Omega\text{m}$
and x varies between 0 and R



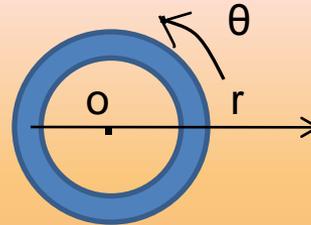
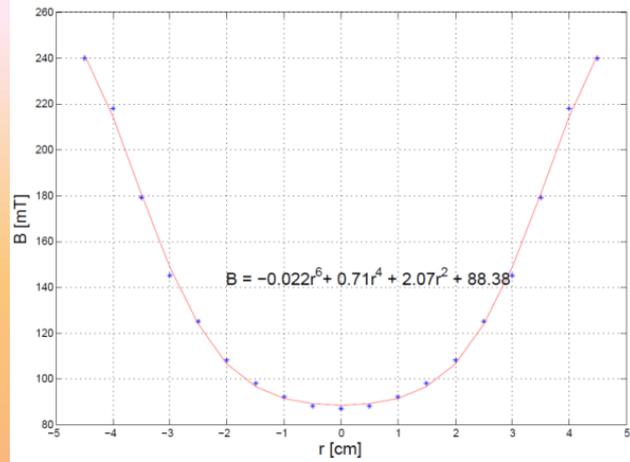
The measured resistance agrees perfectly with the theoretical estimation leading to

$$R = 0.27 \Omega$$

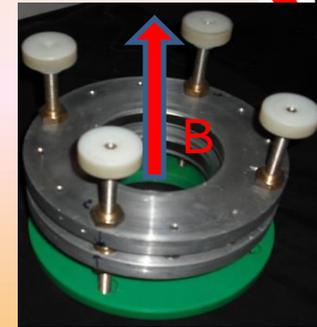
Magnetic field



Two axially magnetized Neodymium Ring magnets.
For a 7 mm separation, the radial magnetic field is:

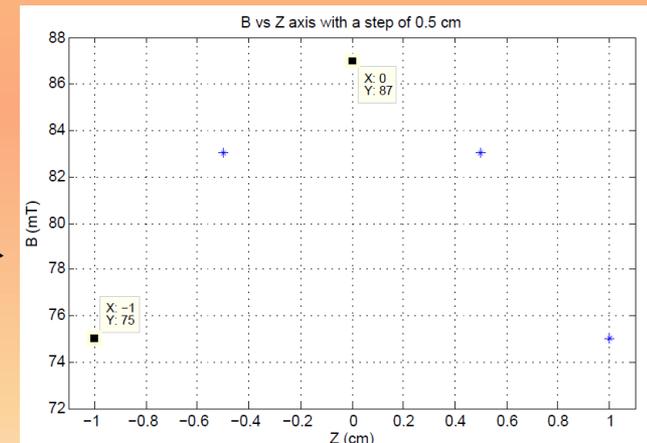
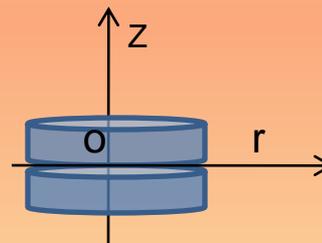
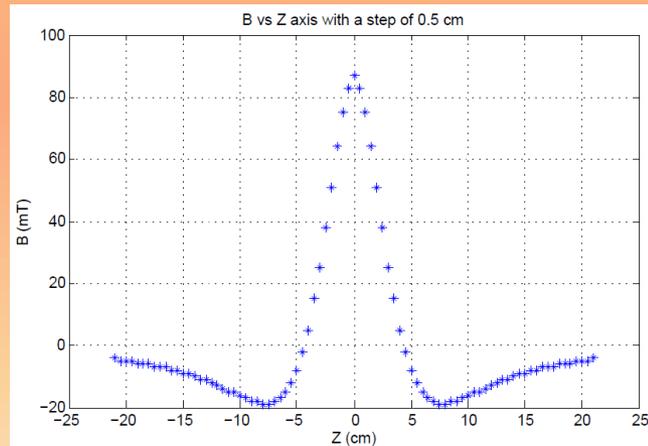


$$B_{\text{center}} = B_{\text{min}} = 87 \text{ mT}$$



The plexy glass cylinder bottom being at $Z = 0$, then points at $Z = 1 \text{ cm}$ suffer a decrease of 14% only.

The axial magnetic field is:



The Coupling of \mathbf{E} and \mathbf{B} creates the driving $\mathbf{J} \times \mathbf{B}$ force which is a vital factor in Navier's Stoke equation describing the flow of liquid gallium



Governing equations of the Fluid dynamics

- The fluid gallium is driven to circulate by the EM force $\mathbf{J} \times \mathbf{B}$
- This contributes strongly in the equations describing the motion

- Navier Stokes equation
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \underbrace{\rho(\mathbf{u} \cdot \nabla) \mathbf{u}}_{\text{Inertial force}} = -\nabla P + \underbrace{\mu \nabla^2 \mathbf{u}}_{\text{viscous force}} + \underbrace{\mathbf{J} \times \mathbf{B}}_{\text{EM force}} \quad (1)$$

- Incompressible equation :
$$\nabla \cdot \mathbf{u} = 0 \quad (2) \quad v = 3.22 \times 10^{-7} \text{ m}^2/\text{s}$$

- Ohms Law:
$$\mathbf{J} = \sigma(\mathbf{E} + \underbrace{\mathbf{u} \times \mathbf{B}}_{\text{Induced E}}) \quad (3) \quad \sigma = 3.86 \times 10^6 (\Omega \cdot \text{m})^{-1}$$

- Magnetic induction equation
$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction term}} + \underbrace{\eta \nabla^2 \mathbf{B}}_{\text{Diffusion term}} \quad (4)$$

The dimensionless eq. of (1) and (4), taking $\mathbf{u} = \frac{\mathbf{u}}{u_0}$, with $u_0 =$ average velocity.

The same for $t, P, \nabla, \nabla^2, J$ and B we get :

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \frac{\nabla^2 \mathbf{u}}{R_e} + N_{im} (\mathbf{E} \times \mathbf{B}) + N_{in} \left(\frac{\mathbf{u} \times \mathbf{B}}{R_E} \right) \quad (5)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{\nabla^2 \mathbf{B}}{R_m} \quad (6)$$



MHD Dimensionless parameters

- Dynamic Reynolds number $Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{lu}{\nu}$
- The interaction parameter $N = \frac{EM \text{ forces}}{\text{Inertial forces}} = \frac{\sigma l E B}{\rho u^2} + \frac{\sigma l B^2}{\rho u} = N_{im} + N_{in}$
- Hartmann number $Ha = \frac{EM \text{ forces}}{\text{viscous forces}} = \sqrt{\frac{\sigma E B}{\rho u \nu}} + \sqrt{\frac{\sigma}{\rho \nu}} x B l$
- Magnetic Reynolds number $R_m = \frac{lu}{\eta} = \frac{\text{magnetic induction}}{\text{magnetic diffusivity}}$

with $\eta = \frac{1}{\sigma \mu_0}$ is the diffusivity of liquid gallium



For length scales equal to the distance between two consecutive electrodes and to the radius of the container

$l(cm)$	$u(cm/s)$	$B(T)$	Re	N_{im}	N_{in}	Ha	R_m
1	5	0.35	1553	1.8×10^4	15.5	1.67×10^4	2.4×10^{-3}
5	3	0.2	4658	2.8×10^5	42	3.6×10^4	7.3×10^{-3}

For $N_{im} \gg N_{in}$ and $Re \gg 1$, the viscous term and induced $u \times B$ drop out, giving from Eq. (1)

$$\rho \frac{\partial u}{\partial t} + \rho(u \cdot \nabla)u = -\nabla P + J \times B$$

For $R_m \ll 1$ the magnetic induction can be neglected when compared to magnetic diffusivity. And we can say that $B_{induced} \ll B_{imposed}$.



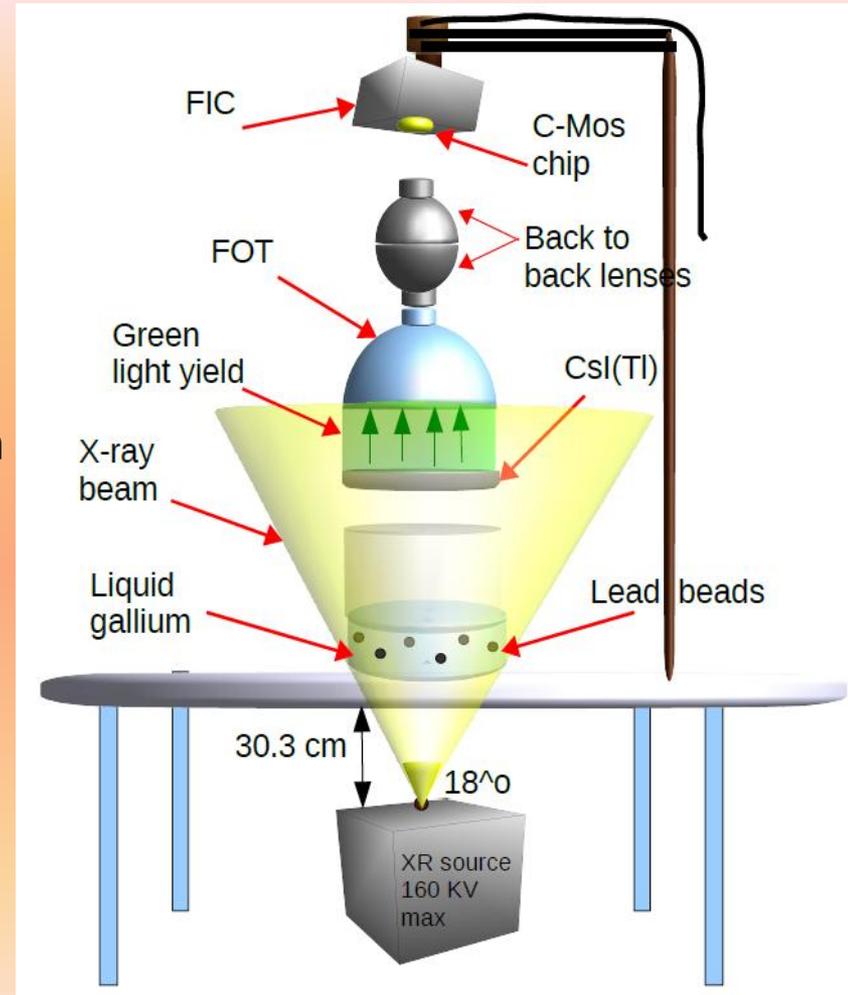
X-ray Imaging Diagnostic

Liquid gallium is opaque, this has created a big challenge to diagnose the flow by techniques like laser Doppler velocimetry and dye traces.

The proposed Fast X-ray Imaging System

The X-ray imaging system contains:

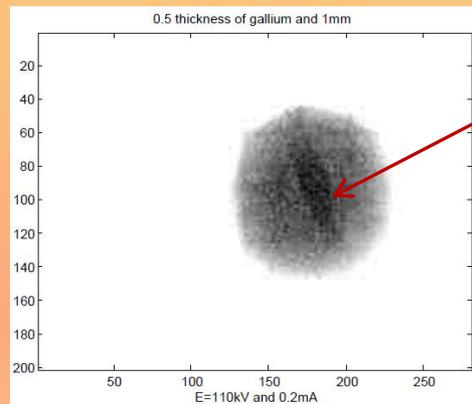
- X-ray source of 160 kV of emerging power through gallium is $P_{\text{output}} = 0.216 \text{ W}$.
- A 200 μm thick CsI(Tl) scintillator of light yield (550 photons/MeV).
- A Fiber Optic taper FOT (4 : 1) magnification ratio and fiber size (25 μm at the larger area)
- Two lenses $f = 12.5 \text{ mm}$, and CMOS chip of pixel size of 144 μm and about a 40 % fill factor area.
- The radiation power detected is 3 mW.



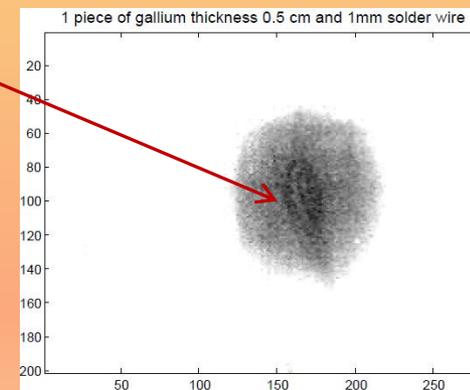
X-ray Imaging of Lead Embedded in Gallium

In collaboration with the AUB medical center Radiation Oncology department, a solder wire of 60/40 that is tin ($Z=50$)/ lead ($Z=82$ and $\rho= 11.34 \text{ g}\cdot\text{cm}^{-3}$) mass ratio is embedded in a 0.5 cm thickness piece of gallium ($Z=31$ and $\rho=5.904\text{g}\cdot\text{cm}^{-3}$).

With an energy of the X-ray source $E = 110 \text{ kV}$ and current beam $I = 0.2 \text{ mA}$, a good contrast in the image was obtained.



Lead and tin



- For this, our technique is based on seeding liquid Gallium heated to $T > 468^\circ \text{C}$ with small beads of diameter 0.5 mm, which work as tracers and serve as good contrast agents observed by the detector.

Assessing the time for beads to float in liquid gallium

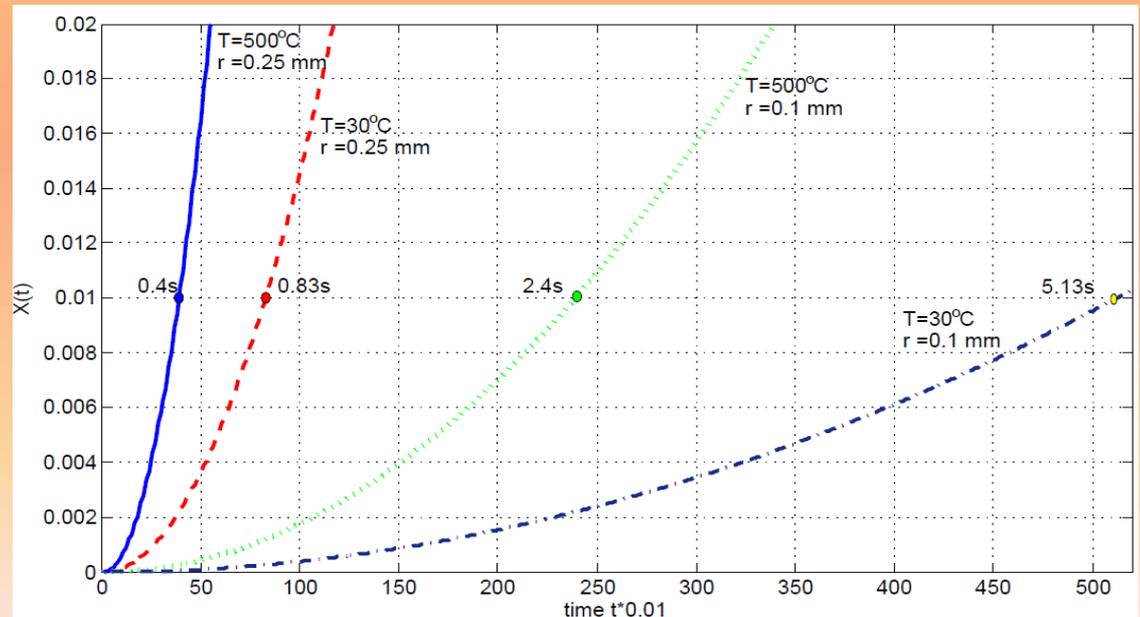


The beads take some time to reach the bottom, during which X-ray imaging must take place.

By Stokes law, the distance verses time is found to be

$$x(t) = \frac{mg}{\xi} \left(1 - \frac{\rho_f}{\rho_b}\right) \left(t + \frac{\xi}{m} \left(e^{-\frac{m}{\xi}t} - 1\right)\right)$$

$\xi = -6\pi\nu(T)r$ is the friction coefficient with m and r are the mass and radius of the bead, and ρ_f and ρ_b are the densities of the fluid and the bead respectively. A plot for $x(t)$ is made for different temperatures T i.e different viscosities and different radii.





Experimental Results: Motion in Liquid Gallium

- Imaging with the Oxidization layer on top of liquid gallium
- Using an imaging Camera which reaches 60 frames/s , without using any imaging technique, beside the normal one.
- In here four electrodes are biased as successively + + - - .
- On the surface, gallium oxide layer is formed assuring that the motion seen is just under it. Some small gallium grains move with the flow under this layer.

Expected force field lines

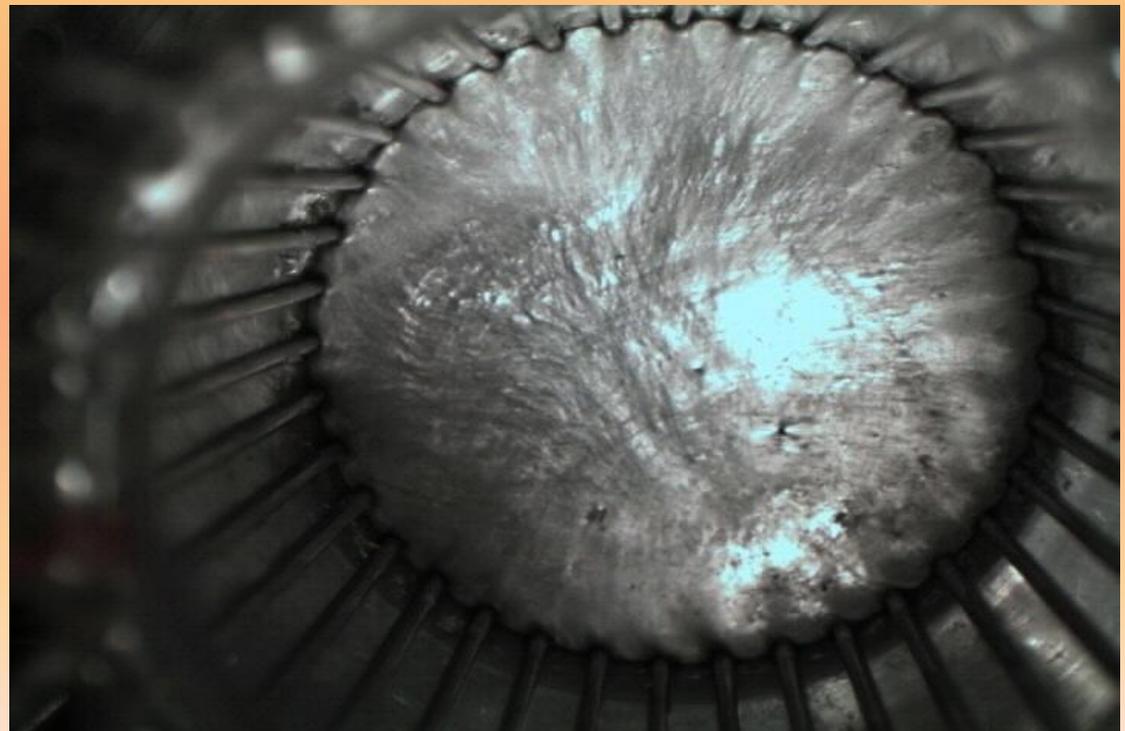
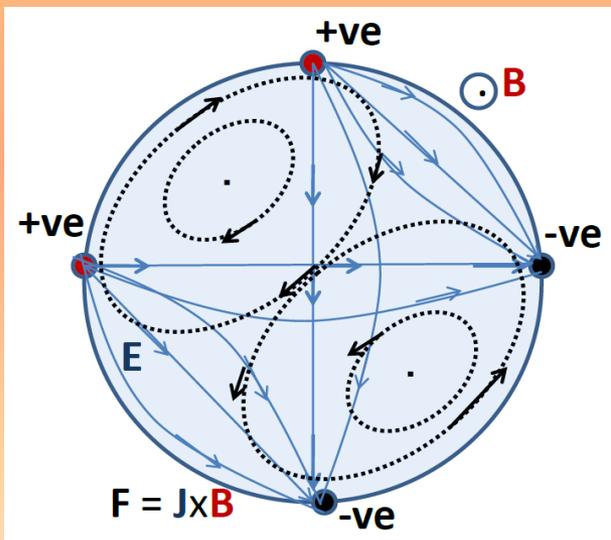
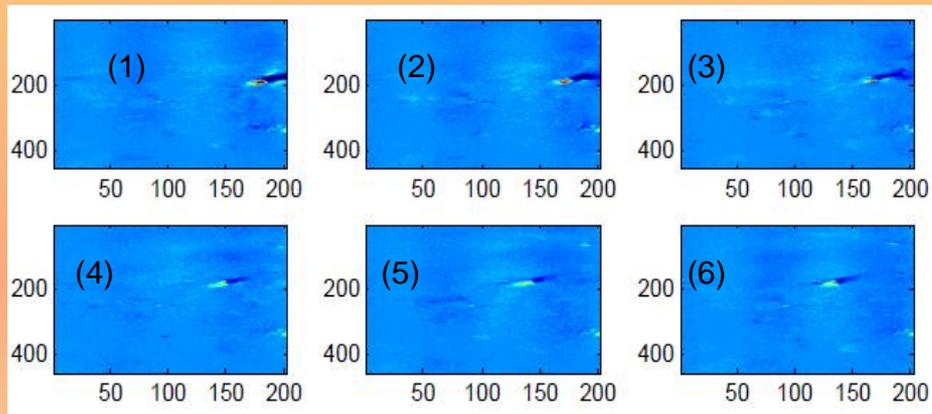
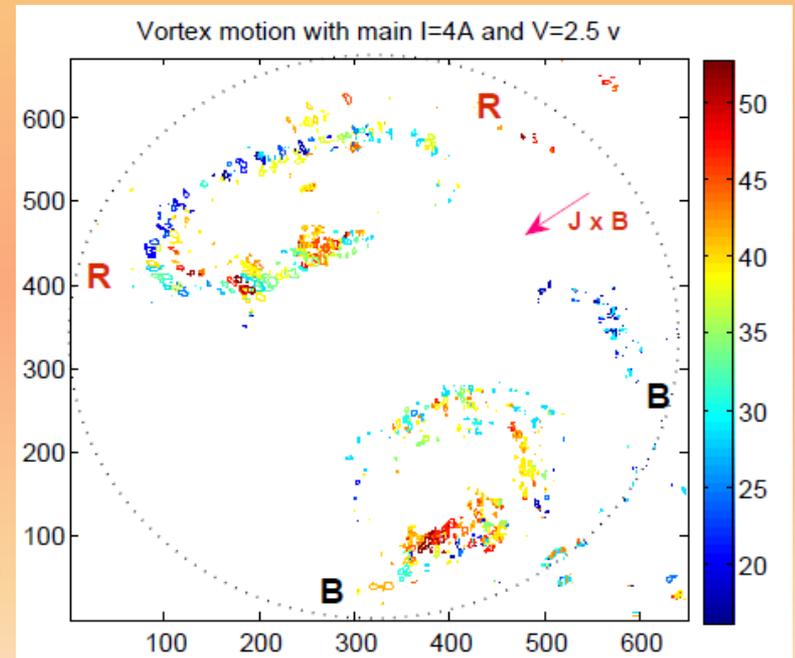


Image analysis of the experimental results

For the ++-- connection, two vortices are expected and traced by the contour plot. As the movie showed before the velocity varies from point to point due to many reasons: dissipation by the surface, change of magnetic field (strongest at the Edges).



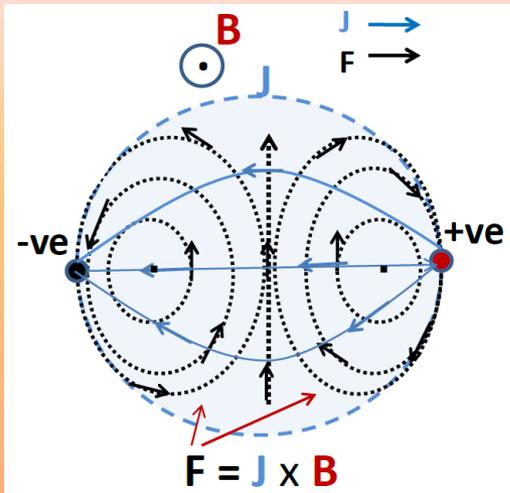
For a gallium grain between the edges and the center the speed is calculated to be $V = 3 \text{ cm/s}$



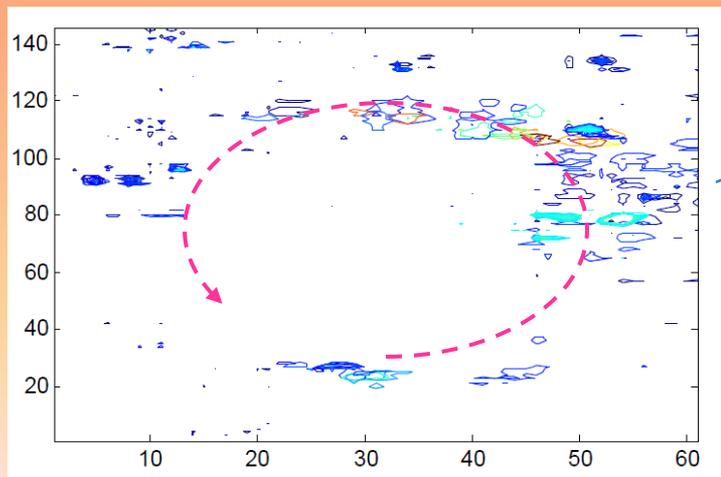
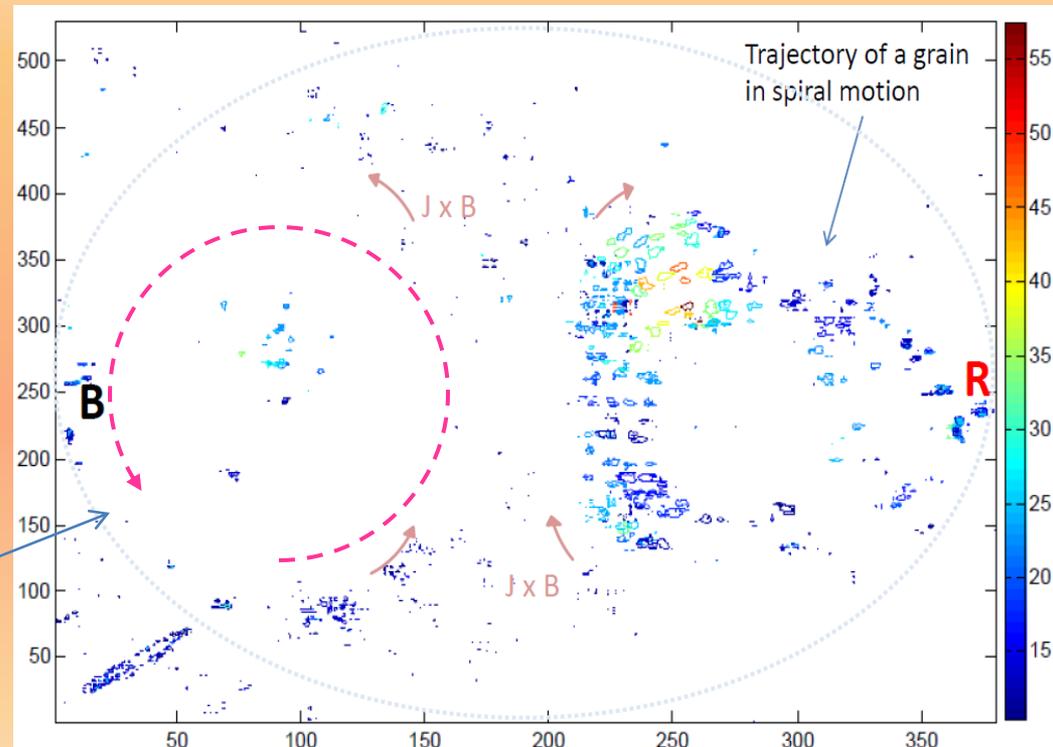
For two electrodes

A contour plot also shows the motion between two electrodes

force field lines as expected



Experimental surface images

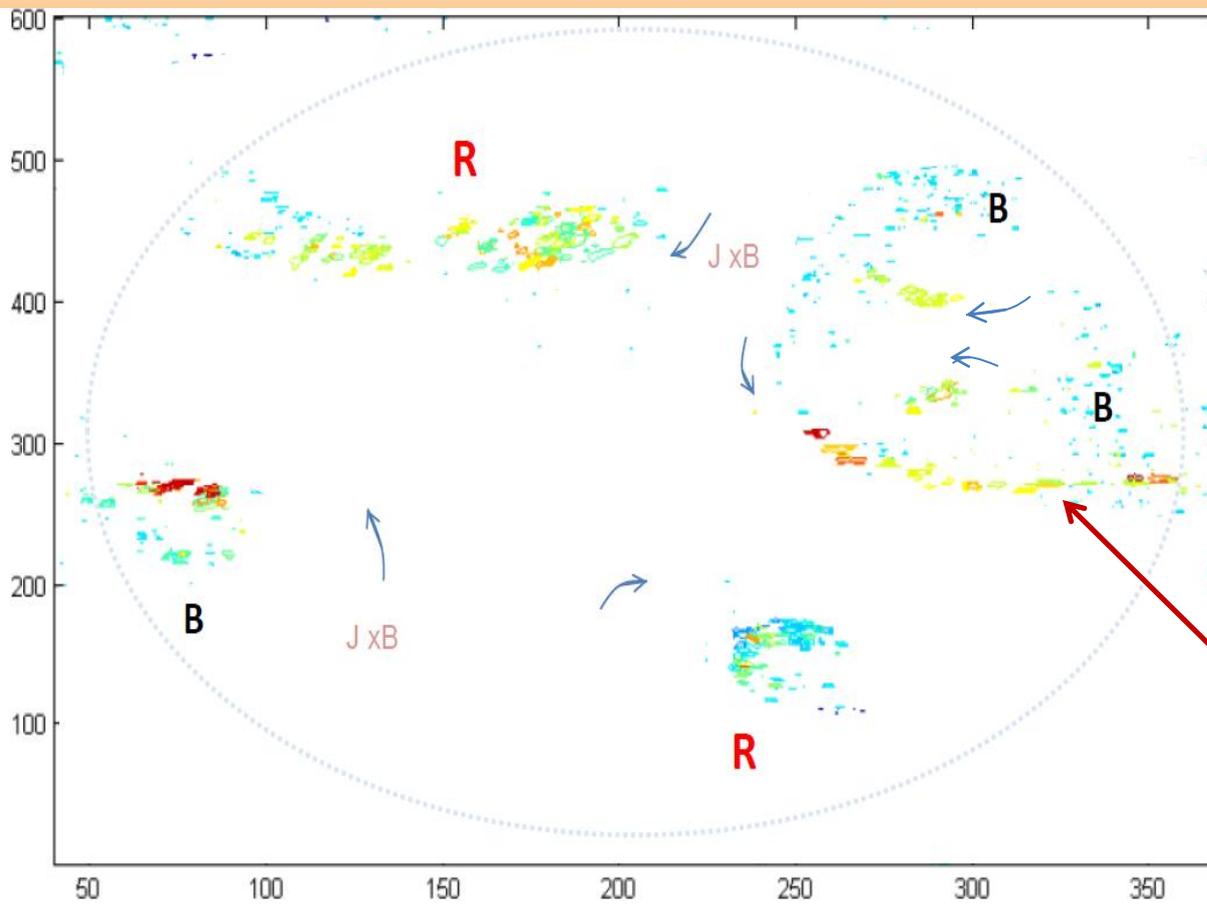
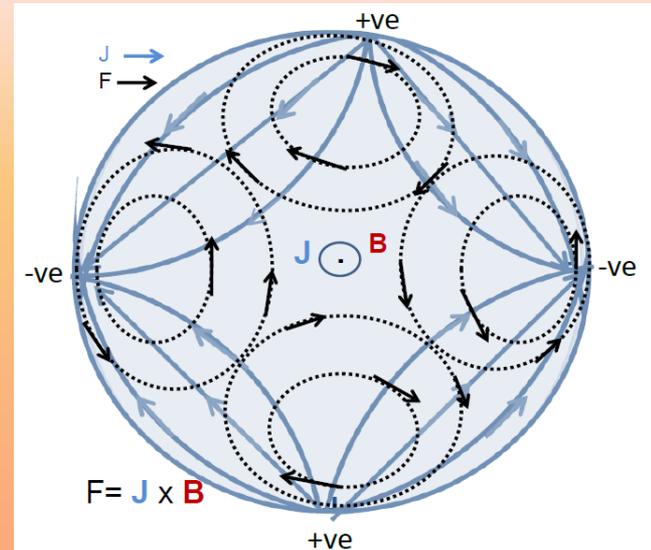


The speed is estimated to be 6 cm/s.

Four electrodes biased +ve -ve +ve -ve

Force field lines as expected

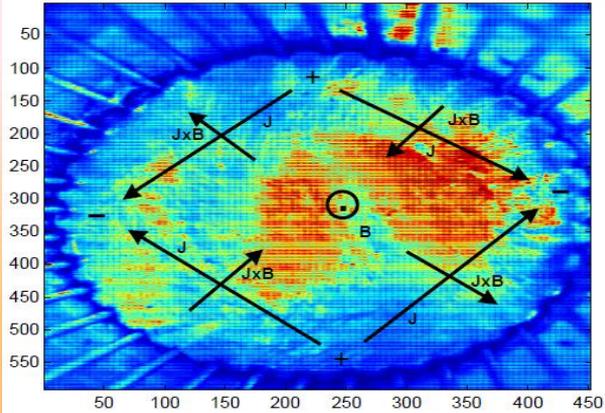
Experimentally found



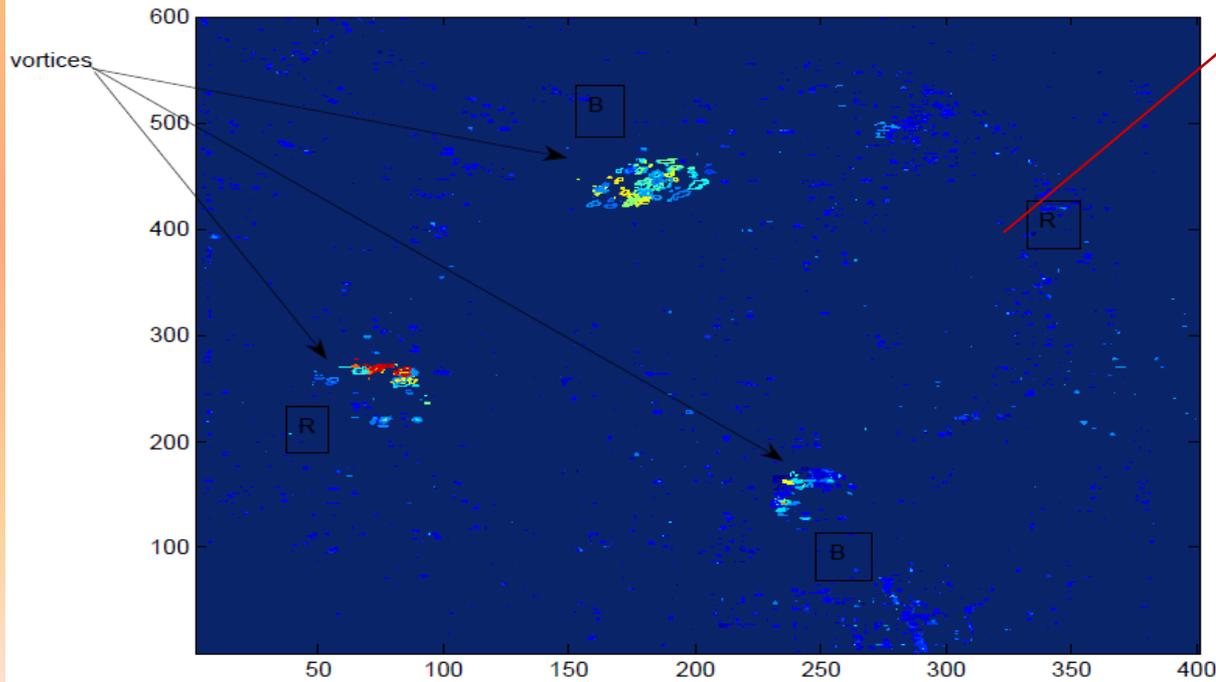
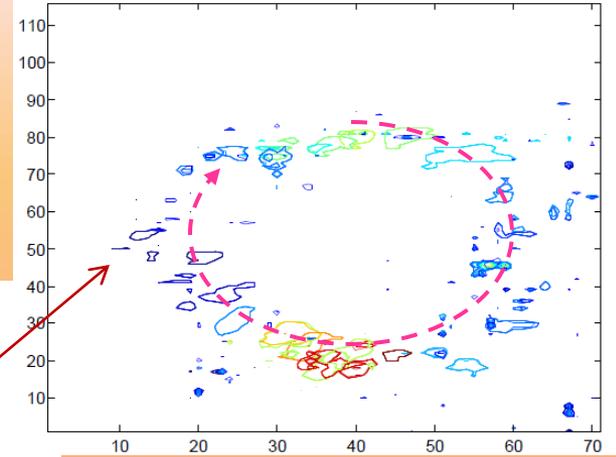
With an extra negative connection shows an extra vortex. For a sufficient current they form a larger vortex surrounding the two.

Four electrodes (+ - + -)

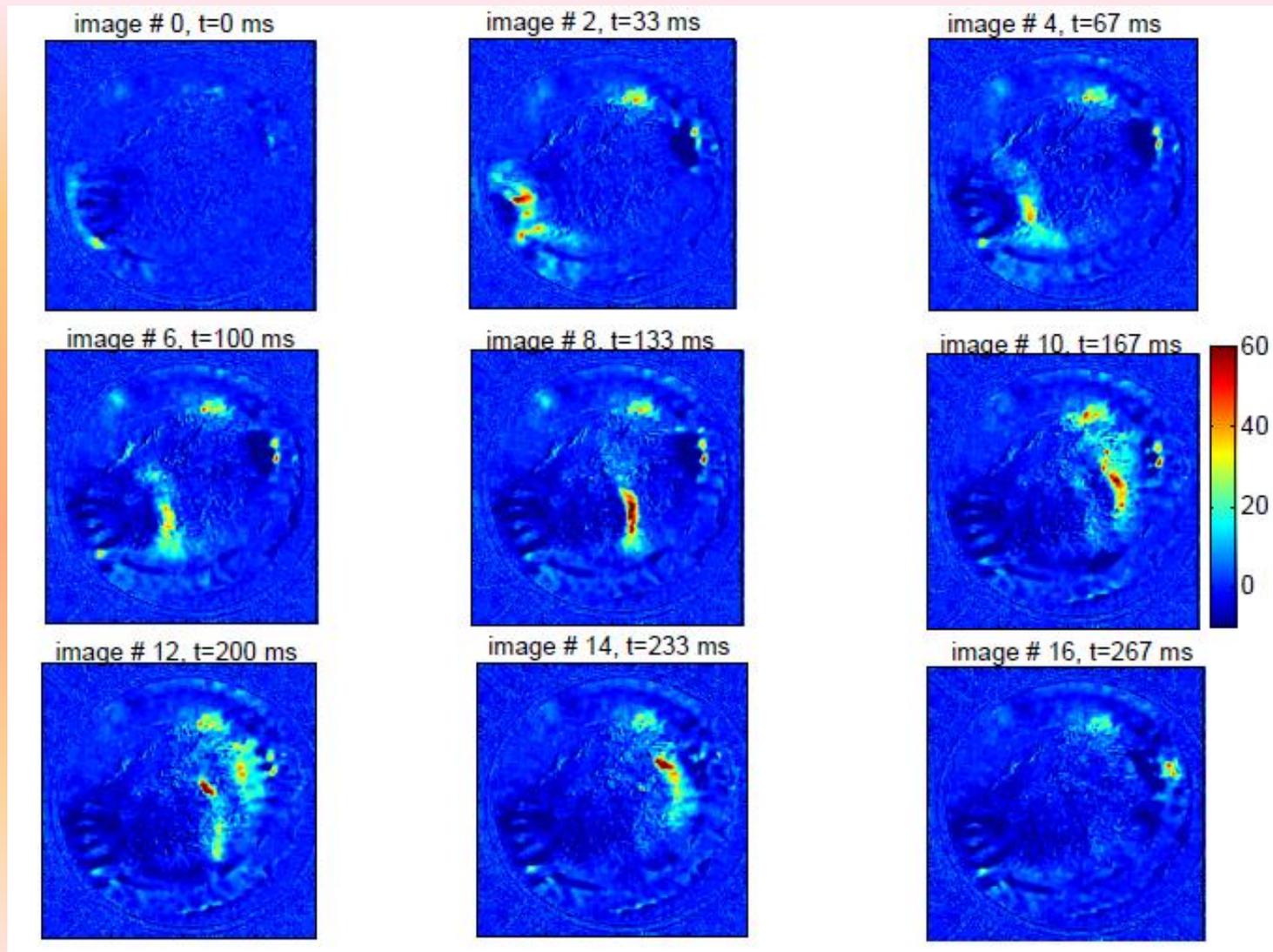
surface of gallium flow when the four electrodes are biased



vortex at an R (+ve) electrode (RBRB)



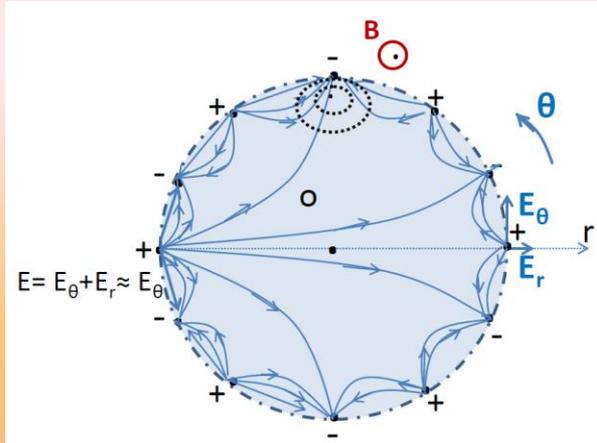
Starting up process when all electrodes are biased alternatively +ve -ve +ve -ve ...



Pulse generation and their radial transport



Electric field lines



$$\rho \frac{\partial u_r}{\partial t} = J_r \times B$$

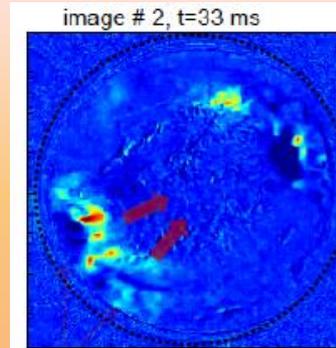
Solving for simplified Navier stoke' s equation and dropping the inertial term (stage of small u), and the pressure term:

$$\rho \frac{\partial \vec{u}}{\partial t} = \vec{J} \times \vec{B}$$

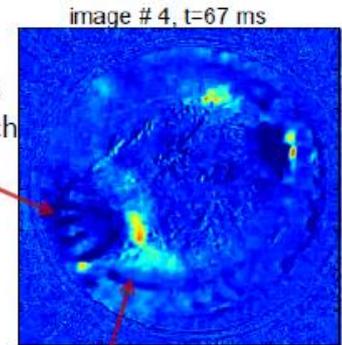
With $\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B})$

Project on r-direction:

$$\rho \frac{\partial u_r}{\partial t} = J_\theta \times B = \sigma(E_\theta + u_r B)$$

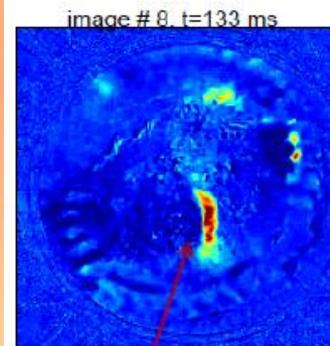


Radial and azimuthally transport of liquid gallium

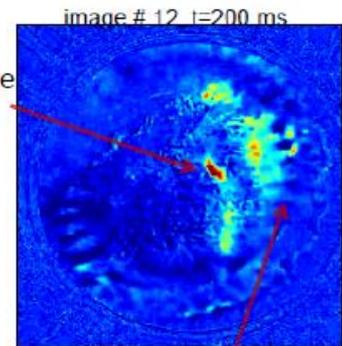


Small vortices formed at each electrode

Combination of the circular pulses



One-line propagation



Another pulse combination follows

Merging of the pulse into the vortices of the opposite electrodes



- Solving the equation gives

$$u_r = u_{r_0} (1 - e^{-\frac{t}{\tau}}) \quad \text{with } \tau = \frac{\rho}{\sigma B^2} = 49ms \quad \text{is the damping time constant.}$$

and $u_{r_0} = \frac{E_\theta}{B}$. That means after $5\tau = 244ms$ the velocity will reach a maximum velocity. Similarly projecting on the θ - direction gives a solution

$$u_\theta = u_{\theta_0} (1 - e^{-\frac{t}{\tau}}) \quad \text{with } u_{\theta_0} = -\frac{E_r}{B}.$$

But since $E_{\theta_0} \gg E_{r_0}$ then $u_r \gg u_\theta$ and $\vec{u} = \vec{u}_r + \vec{u}_\theta \approx \vec{u}_r$

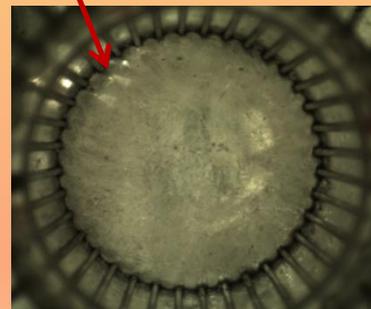
And this explains the motion created at the boundaries which is dominated by the radial direction causing these pulse propagations from one side to other, and damp completely at the frame 16, after

$$t_{\text{exp}} = 267ms \approx 5\tau$$

- **Conclusion :**

- The real motion of the liquid is visualized at the first stages of driving the gallium, and its happening only below the surface.
- For this the oxidizing layer is suppressing the motion and making the detection more difficult.
- This carried us to remove the layer from surface even though making it reflective as a mirror but a more huge motion is detected.

Ga₂O₃ layer



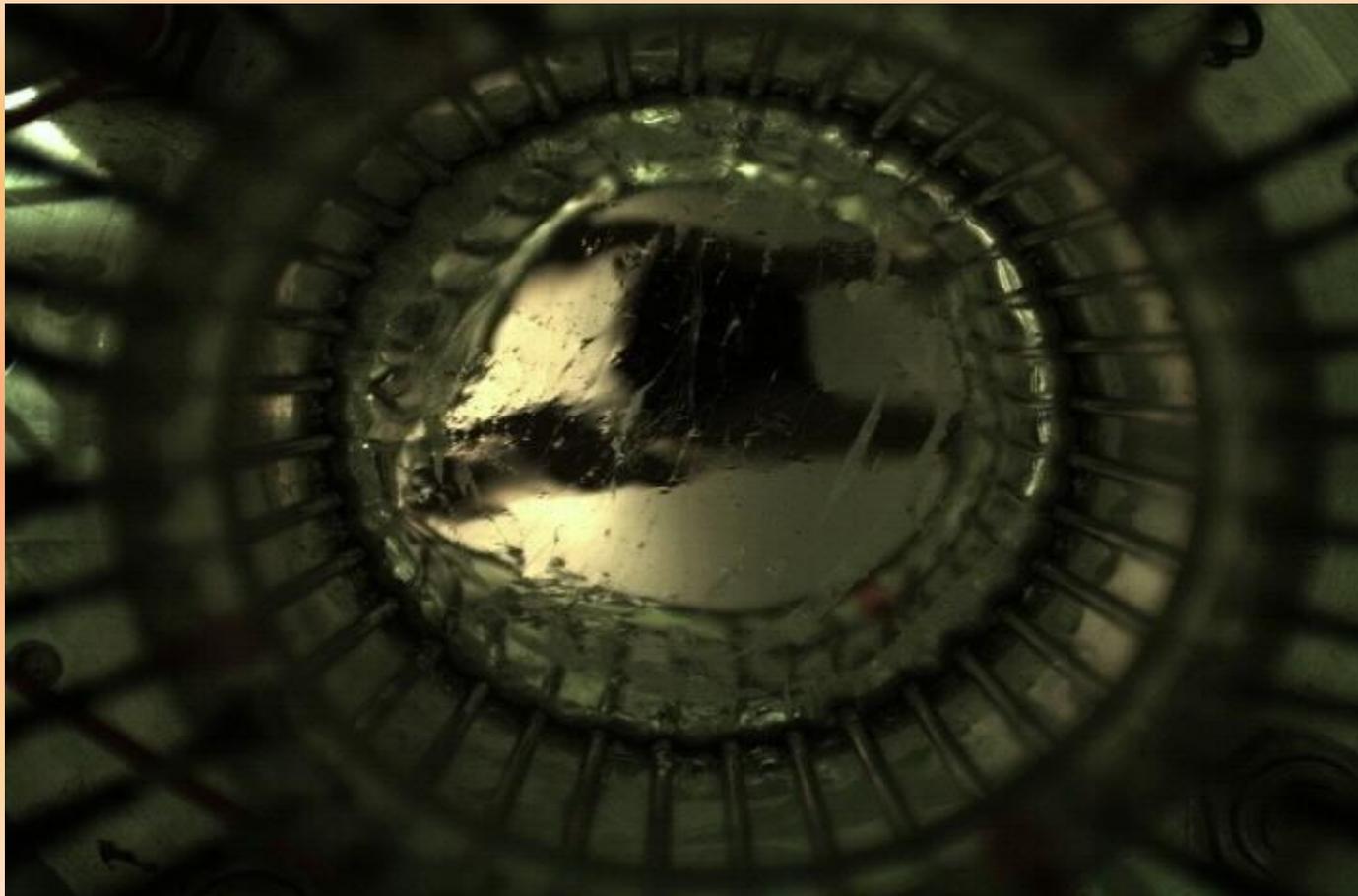
Partially removed





Experimental results after moving the oxidized layer

- A movie for four electrodes working under $I = 10$ A, shows huge motion with somehow propagation of vortices.



Conclusion



- Gallium oxidizes with oxygen, making the visualization harder as it also suppresses the motion on the surface.
- It has a high surface tension that also suppresses the motion on the surface. For this the surface measurements give a rough idea about the bulk motion.
- Recent studies has proved that strong magnetic fields suppress motion in planes perpendicular to them, this is one reason for the motion to form coherent vortices at some periods of time, even though boundary layers are small enough predicting turbulence.
- A solution for some of problems is Gallium indium tin.



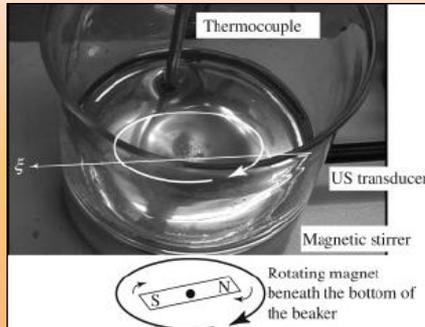
Future work

- Building the X-ray imaging system, thus visualizing the bulk to determine the velocity and vorticity of motion.
- Using Gallium Indium tin utilizing the laser reflection technique to determine the motion of the surface flow.

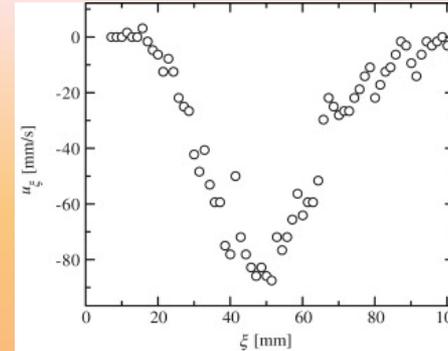
Ultrasound Imaging

- Ultrasound velocity Profiling (Yokosoka Japan):**

utilizing Doppler shift frequency and ultrasonic ecography to determine an instantaneous velocity profile.



$B = 0.2 \text{ T}$, US frequency = 4 MHz



instantaneous velocity profile where ξ represents the distance from the ultrasonic transducer

A small experiment in the AUB medical center didn't show promising results even though in previous studies it is. For this the trial for the technique is dropped.

- Laser reflection measurement:**

Inspired by an experiment done by Hantao (Princeton University), who measured the damping of a transverse wave on the surface of gallium. The same concept could be used to measure the inclination on the surface and in turn the displacement. This has been our recent trial in the laboratory.